THE UNIVERSITY OF MELBOURNE

Department of Economics

Working Paper Series

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August 2009

Research Paper Number 1081

ISSN: 0819-2642
ISBN: 978 0 7340 4434 1

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Welfare-Reducing Mergers in Differentiated Oligopolies with Free Entry[1]

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April 2009

[1]We would like to thank Federico Etro, Toshihiro Matsumura, participants of the conference on Endogenous Market Structures and Industrial Policy (Milan, 2008) for valuable discussion, and Christian Roessler for his help with the numerical analysis.
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Abstract

Antitrust authorities regard the possibility of post-merger entry and merger-generated efficiencies as two factors that may counteract the negative effects of horizontal mergers. This paper shows that in differentiated oligopolies with linear demand, all entry-inducing mergers harm consumer welfare. This is because if there is entry following a merger, it implies that the merger-generated efficiencies were not sufficiently large. Mergers which induce exit, due to sufficiently high cost savings, always improve consumer welfare.

JEL codes: L13, L22, L41, K21

Keywords: Horizontal mergers; product differentiation; entry; cost efficiencies; antitrust policy

1 Introduction

Antitrust authorities regard the possibility of post-merger entry and merger-generated efficiencies as two factors that may counteract the negative effects of horizontal mergers. For instance, it is stated in the Merger Guidelines of the Australian Consumer and Competition Commission (ACCC) that "a credible threat of new entry alone may prevent any attempt to exercise market power in the first place."[1] Similar statements appear in the Horizontal Merger Guidelines issued by the antitrust authorities in the United States and the European Commission's horizontal merger guidelines.[2]

If barriers to entry are low, one would expect a price increase by the merging firms to invite entry, which would push prices back to their pre-merger level. Hence, consumers may be no worse off than they were before the merger.[3] This intuition has been shown to hold in a model with homogeneous products and Cournot competition by Davidson and Mukherjee (2007). They find that the long-run effect of any merger on the price level and, hence, on consumer welfare is zero.[4,5]

[1] See paragraph 7.17 in the ACCC Merger Guidelines available at www.accc.gov.au/content/index.phtml/itemId/809866.

[2] See Sections 3 and 4 of the Horizontal Merger Guidelines, issued jointly by the Department of Justice (DOJ) and Federal Trade Commission (FTC) in the United States (US). Shapiro (1996) presents a summary of the US antitrust authorities' approach to mergers in differentiated industries. For the approach in Europe, see Sections VI and VII of the Guidelines on the assessment of horizontal mergers under the Council Regulation on the control of concentrations between undertakings, Official Journal of the European Union (2004/C 31/03).

[3] Many proposed mergers are either challenged or allowed based on arguments involving entry. For example, in the US, the FTC argued that entry was unlikely to prevent the anticompetitive effects arising from the proposed merger between Staples and Office Depot. In contrast, entry was the main reason the DOJ decided not to challenge the proposed merger between National Oilwell and Varco. See the "Commentary on the Horizontal Merger Guidelines," issued jointly by the DOJ and FTC in 2006, for a summary of these cases and many others where entry considerations played an important role (www.usdoj.gov/atr/public/guidelines/215247.htm). Similarly, in Europe, the proposed mergers between Agfa-Gevaert and DuPont (Case No IV/M.986), Hitachi and IBM (Case No COMP/M.2821), and HP and Compaq (Case No COMP/M.2609) were not challenged after the Commission investigated the possibility of post-merger entry.

[4] Spector (2003) also analyzes the effects of mergers with entry in a model with homogeneous products and Cournot competition. Allowing for heterogeneity in costs across firms, he finds that without cost synergies, all profitable mergers must harm consumer welfare. In contrast, Davidson and Mukherjee (2007) assume that all firms are symmetric to start with and show that there are no profitable mergers without cost synergies. Hence, the set of mergers Spector (2003) analyzes is empty in their model.

[5] An extensive literature examines the impact of mergers in models without entry. See, for example, Deneckere and Davidson (1985), Farrell and Shapiro (1990), and, more recently, Kao andMenezes (2007).

In this paper, we show that this result and the logic behind it does not readily extend to the more realistic case of differentiated products. The impact of entry in markets with differentiated products differs from its impact in markets with homogeneous products in two important ways. First, when products are imperfect substitutes, merger-induced entry puts less competitive pressure on the existing firms in the market than it does when products are perfect substitutes. Second, in a framework with product differentiation, entrants provide additional variety, which is valuable to consumers. The implication of these two effects for the net impact of mergers on consumer welfare is not obvious *a priori*.

Our results reveal that mergers which induce entry always harm consumer welfare while mergers which induce exit always improve consumer welfare. This is because if there is entry following a merger, it implies that the merger-generated efficiencies were not sufficiently large. Merger-generated efficiencies determine how much the merging firms raise their prices and invite entry. Hence, in a model with free entry and exit, mergers are harmful unless they result in sufficiently high cost savings.

Two related papers which also analyze the effects of mergers in a set-up with differentiated products and entry are Werden and Froeb (1998) and Cabral (2003).[6] Werden and Froeb (1998) analyze the entry-inducing effects of mergers using simulations in a set-up with highly-concentrated markets (no more than 8 incumbents), logit demand, and Bertrand competition. They consider mergers involving two firms and assume that there is a single potential entrant. They conclude that firms proposing to merge must expect either to achieve significant efficiency gains or not to induce entry. Hence, there may be no need for courts to consider entry explicitly because it collapses into efficiency considerations. In contrast, we find that entry will take place in cases when the merger-generated efficiencies are not high enough to prevent consumers from being harmed by the merger. Investigating the combined effect of entry and cost efficiencies, we reach the conclusion that entry-inducing mergers may be profitable and may take place even though they are harmful for consumers.

[6] See also Gowrisankaran (1999) for an analysis of a dynamic Cournot game with endogenous investment, merger, entry and exit decisions. His computational analysis suggests that mergers' anticompetitive effects are unlikely to be reversed by entry.

Using the Salop model of product differentiation, Cabral (2003) shows in the specific case of two firms merging to form a monopoly that entry improves consumer welfare and cost efficiencies may even harm consumer welfare. The assumption of merging to monopoly is crucial for his results because if there is a monopoly in the Salop model, consumers do not benefit from the cost efficiencies at all. Hence, sufficiently large cost efficiencies may hurt consumers because they have the effect of deterring entry only. In contrast, using a linear demand system, we are able to consider mergers of any size, and analyze the combined effect of entry and cost efficiencies. This allows us to show that although cost efficiencies affect entry adversely, their impact on welfare dominates and consumer welfare increases in the level of cost efficiencies.

We proceed in the next section by analyzing the consumer welfare effects of mergers in a set-up with Cournot competition and "non-drastic" mergers, where there exists a positive number of outsider firms in the market in equilibrium. We then discuss, in Section 3, how the results in Section 2 extend to the case of Bertrand competition and drastic mergers.

2 Model and results

Consider a model where the representative consumer's utility function is given by

$$U\left(q_0; q_i, i \in \{1, ..., N\}\right) = \alpha \sum_{i=1}^{N} q_i - \frac{\beta}{2} \sum_{i=1}^{N} q_i^2 - \frac{\gamma}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} q_i q_j + q_0, \tag{1}$$

where $\alpha > 0$ and $\beta > \gamma > 0$. q_i stands for the quantity of variety i, q_0 stands for the quantity of the numeraire good, and α is a measure of the size of the market for the differentiated good. The inverse demand function for good i is given by

$$p_i\left(q_i, \boldsymbol{q}_{-i}\right) = \alpha - \beta q_i - \gamma \sum_{j \neq i}^{N} q_j \tag{2}$$

where $\boldsymbol{q}_{-i}$ denotes a vector of quantity choices by firm i's rivals.

Producers of different varieties compete by choosing quantities. Hence, firm i chooses

q_i, taking the output choices of its rivals as given, to maximize

$$\pi_i\left(q_i, \boldsymbol{q}_{-i}; c_i, F_i\right) = q_i\left(\alpha - \beta q_i - \gamma \sum_{j \neq i}^{N} q_j - c_i\right) - F_i, \tag{3}$$

where c_i and F_i are the constant marginal and fixed cost of production for variety i, respectively. There is a large (infinite) number of potential entrants into the market. We assume that initially $c_i = c$ and $F_i = F$ for all i and the market is in a free entry equilibrium. This implies

$$\pi^*\left(q^*, N^*; c, F\right) = q^*\left[\alpha - \beta q^* - \gamma\left(N^* - 1\right) q^* - c\right] - F = 0, \tag{4}$$

where q^* is the quantity that maximizes firm i's profits when all other firms produce q^* and N^* is the number of firms for which all firms earn zero profits.[7]

To evaluate the effects of horizontal mergers, we assume that an exogenous group of $M \leqslant N^*$ firms merge. The merged firms continue to produce the same products, but incur constant marginal cost of $c^m \leqslant c$ and per-variety fixed cost of $F^m \leqslant F$ due to merger-specific synergies. Following the merger, the potential entrants to the market simultaneously decide whether or not to enter and the incumbents in the market which are not part of the merger simultaneously decide whether or not to exit. After the potential entrants and the incumbent outsiders make their entry and exit decisions, all firms compete by simultaneously choosing their quantities. The merging firms maximize their joint profits while the outsiders maximize individual profits.

We let N^m stand for the equilibrium number of firms in the market after the merger, which is determined by the zero-profit condition after the merger. Using q^m and q^o to denote the per-firm production level of the merged firms and the outsiders, respectively, our first result is the following.

Lemma 1 *The outsiders produce the same amount both with and without the merger (i.e., $q^o = q^*$). Moreover, total quantity produced of the differentiated goods is also the same both with and without the merger: $Mq^m + \left(N^m - M\right) q^o = N^* q^*$.*

[7] Note that since $\frac{\partial^2 \pi_i\left(q_i, \boldsymbol{q}_{-i}; c_i, F_i\right)}{\partial q_i^2} + \sum_{j \neq i} \frac{\partial^2 \pi_i\left(q_i, \boldsymbol{q}_{-i}; c_i, F_i\right)}{\delta q_j \partial q_i} < 0$ and the maximized value of firm i's profit function for a given N is decreasing in N, the equilibrium is unique.

Proof. Let $\sigma_i = \sum_{j \neq i}^{N} q_j$. Each outsider firm i solves

$$\max_{q_i} \quad \pi_i \left(q_i, \sigma_i; c, F\right) = q_i \left[\alpha - \beta q_i - \gamma \sigma_i - c\right] - F \tag{5}$$

before and after the merger. Let $\widehat{\pi}_i \left(\widehat{q}_i \left(\sigma_i\right), \sigma_i; c, F\right)$ stand for the maximized value of this profit function for given σ_i. Applying the envelope theorem, it is clear that $\frac{d\widehat{\pi}_i(\widehat{q}_i(\sigma_i),\sigma_i;c,F)}{d\sigma_i} < 0$. Since an outsider firm i earns the same profits with and without the merger (which is equal to zero), it must be the case that it faces the same value of σ_i with and without the merger. However, if this is the case, it must also be true that the same value of q_i maximizes its profits with and without the merger. Hence, we have $q^o = q^*$.

Since an outsider firm faces the same value of σ_i with and without the merger, substituting for q^o and q^m in the expression for σ_i gives us

$$Mq^m + \left(N^m - M - 1\right) q^* = \left(N^* - 1\right) q^*. \tag{6}$$

Finally, adding q^* to both sides yields

$$Mq^m + \left(N^m - M\right) q^* = N^* q^* \tag{7}$$

and completes the proof. ■

Using (7) we can derive the relationship between the number of firms with and without the merger.

$$N^m - N^* = M \frac{\left(q^* - q^m\right)}{q^*}. \tag{8}$$

This implies that there are more firms in the market with the merger if and only if the merging firms produce less with the merger than they do without it.

We are now in a position to evaluate the effect of the merger on consumer welfare. Letting y stand for the income level and using the fact that $q^0 = y - \sum_i q_i p_i = y - \sum_i q_i \left(\alpha - \beta q_i - \gamma \sum_{j \neq i}^{N} q_j\right)$, we can write the utility function as

$$U^* \left(q^*, N^*; y\right) = y + \frac{\beta}{2} N^* \left(q^*\right)^2 + \frac{\gamma}{2} N^* \left(N^* - 1\right) \left(q^*\right)^2 \tag{9}$$

without the merger, and as

$$U^m(q^m, N^m; y) = y + \frac{\beta}{2}\left[M(q^m)^2 + (N^m - M)(q^*)^2\right] + \frac{\gamma}{2}\left[\begin{array}{c} 2M(N^m - M)q^m q^* \\ +(N^m - M)(N^m - M - 1)(q^*)^2 + M(M-1)(q^m)^2 \end{array}\right] \tag{10}$$

with the merger. Subtracting (9) from (10) gives

$$U^m(q^m, N^m; y) - U^*(q^*, N^*; y) = \frac{(\beta - \gamma)}{2} M q^m (q^m - q^*). \tag{11}$$

Hence, consumer welfare increases as a result of the merger if and only if $q^m > q^*$, which from (8) is equivalent to $N^m < N^*$.[8] The following proposition summarizes this result.

Proposition 1 *If a merger induces entry, it decreases consumer welfare and if it induces exit, it increases consumer welfare. If the number of firms does not change as a result of a merger, the merger has no impact on consumer welfare.*

We next show that the amount of merger-induced entry and exit depends on the level of merger-generated efficiencies. This allows us to establish that if a merger does not generate any marginal cost savings (i.e., if $c^m = c$), we have $q^m < q^*$ and the merger harms consumer welfare. Conversely, if it generates sufficiently large marginal cost savings, it must improve consumer welfare.

Proposition 2 *There exists a $\widehat{c} < c$ such that $q^m = q^*$ when $c^m = \widehat{c}$. A merger harms consumer welfare if $c^m > \widehat{c}$ and improves consumer welfare if $c^m < \widehat{c}$.*

Proof. The first order conditions for the merged firm's and an outsider firm's maximization problems are given by

$$\alpha - 2(\beta + \gamma(M-1))q^m - \gamma(N^m - M)q^o - c^m = 0 \tag{12}$$

and

$$\alpha - (2\beta + \gamma(N^m - M - 1))q^o - \gamma M q^m - c = 0. \tag{13}$$

[8] Note that if $\beta = \gamma$, we have homogeneous products and get the same result as in Davidson and Mukherjee (2007).

In equilibrium, the following zero-profit condition must also be holding.

$$q^o \left[\alpha - (\beta + \gamma (N^m - M - 1)) q^o - \gamma M q^m - c\right] - F = 0. \tag{14}$$

The critical c^m value, $\widehat{c}$, can be found by solving (13) and (14) simultaneously for q^o and N^m and then substituting for these values in (12) after setting $q^m = q^o$. Solving the resulting expression for c^m yields

$$\widehat{c} = c - \gamma (M - 1) \sqrt{\frac{F}{\beta}}, \tag{15}$$

which is clearly less than c. It is > 0 for sufficiently low values of γ, M and F, and for sufficiently high values of β.

We next show that $\frac{dq^m}{dc^m} < 0$, which, combined with (11), proves the result stated in the proposition. The equilibrium values of q^m, q^o and N^m can be found by solving (12), (13) and (14) simultaneously. It is straightforward to show that

$$\frac{dq^m}{dc^m} = -\frac{1}{2\beta + \gamma (M - 2)} \tag{16}$$

which is < 0 because $\beta > \gamma > 0$. ■

The amount of merger-generated efficiencies determines the extent to which the merging firms find it profitable to raise their prices and, hence, invite entry. Mergers have no negative impact on consumer welfare in cases when the level of cost savings are such that either the number of firms in the market does not change or there is exit. In fact, mergers which cause exit always improve consumer welfare. Although it seems natural that the consumer welfare harm caused by a merger should be decreasing in the resulting marginal cost savings, the results stated above are surprising for two reasons. First, they contradict the conventional wisdom that low barriers to entry are a sufficient condition for mergers not to hurt consumer welfare. Second, they imply that with the merger, if consumers consume the same total quantity spread over a larger variety of products, they are worse off. If they spread it over a smaller variety, they are better off. The reason is that consumers have to pay too much for the additional variety that the merger induces and this decreases the impact of entry.

To determine the role for policy, we next explore the link between c^m and the profitability of mergers.

Proposition 3 *If the merger-generated efficiencies are such that* $F^m \leqslant F$ *and* $c^m \leqslant \widehat{c}$*, the merger is strictly profitable.*

Proof. Consider first the case when $F^m = F$ and $c^m = \widehat{c}$. From the definition of $\widehat{c}$, we know that $q^m = q^*$ and $N^m = N^*$. At this point, the per-firm payoff for the merged firms,

$$q^* \left[\alpha - \beta q^* - \gamma \left(\left(N^* - 1\right) q^*\right) - \widehat{c}\right] - F, \tag{17}$$

is clearly greater than the payoff of an outsider firm,

$$q^* \left[\alpha - \beta q^* - \gamma \left(\left(N^* - 1\right) q^*\right) - c\right] - F = 0, \tag{18}$$

since $\widehat{c} < c$ as established in (15). Hence, the merger is strictly profitable.

For $c^m < \widehat{c}$, we use the envelope theorem to get

$$\begin{aligned} \frac{d\pi^m \left(q^m, q^o, N^m; c^m, F^m\right)}{dc^m} &= \frac{\partial \pi^m \left(q^m, q^o, N^m; c^m, F^m\right)}{\partial c^m} \\ &\quad + \frac{\partial \pi^m \left(q^m, q^o, N^m; c^m, F^m\right)}{\partial N^m} \frac{\partial N^m}{\partial c^m} \end{aligned} \tag{19}$$

since $\frac{\partial q^*}{\partial c^m} = 0$. $\frac{\partial \pi^m (q^m, q^o, N^m; c^m, F^m)}{\partial c^m}$ and $\frac{\partial \pi^m (q^m, q^o, N^m; c^m, F^m)}{\partial N^m}$ are clearly negative. The second part of the second term on the right hand side can be written as

$$\frac{\partial N^m}{\partial c^m} = \frac{\partial N^m}{\partial q^m} \frac{\partial q^m}{\partial c^m}. \tag{20}$$

We know from (8) and (16) that the two terms on the right hand side are negative. This implies that $\frac{d\pi^m (q^m, q^o, N^m; c^m, F^m)}{dc^m} < 0$ and the result follows. ■

Propositions 2 and 3 imply that in a free entry environment, all mergers which improve consumer welfare are profitable. This implies that all mergers which improve social welfare are also profitable since the outsiders make zero profits with free entry. Of course, the converse is not true. From the continuity of the profit function, we know that some mergers which harm consumer and/or social welfare are still profitable.

3 Concluding remarks

In the analysis of the effects of mergers, it is important to consider the impact of merger-generated efficiencies on the possibility of post-merger entry. Although antitrust authorities consider both merger-generated efficiencies and entry as factors which may counteract the negative effects of horizontal mergers, the link between them is not emphasized in their guidelines. In a model with endogenous entry and differentiated products, we have shown that the net effect on consumer welfare is dominated by cost efficiencies in the sense that consumer welfare is increasing in the level of cost efficiencies. Mergers which do not generate any cost savings strictly hurt consumer welfare. It is not necessarily desirable to have more entry because more entry following a merger implies that the merger-generated efficiencies were not sufficiently large. In fact, mergers which induce entry always harm consumer welfare while mergers which induce exit always improve consumer welfare.

Our analysis has been based on some specific assumptions. We conclude by considering some extensions to the model we presented. First, an immediate question which arises is to what extent these results hold if the firms are price-setters. Although the analysis of the consumer welfare effects of the merger becomes intractable with Bertrand competition, we have conducted numerical analysis with a large range of parameters to verify that the results stated above are not specific to the case of Cournot competition. The results from the numerical analysis reveal that mergers increase consumer welfare in cases where the amount of merger-generated efficiencies is such that the merging firms lower their price as a result of the merger (i.e., if $p^m < p^*$). However, if the merging firms increase their price as a result of the merger, consumer welfare decreases. We show in the Appendix that there are more firms in the market with a merger (i.e., $N^m > N^*$) if the merging firms increase their price after the merger (if $p^m > p^*$). This implies, together with the results from the numerical analysis that, as in the case of Cournot competition, mergers which induce exit (due to sufficiently large efficiencies) improve consumer welfare, while mergers which induce entry (due to a relatively small level of efficiencies) decrease consumer welfare.

Second, the analysis has been based on the assumption that the mergers are non-drastic.

That is, we have assumed that the merger-generated cost savings are such that with the merger there are always some outsider firms which find it profitable to remain in the market. The model extends naturally to the case of drastic mergers, where $N^m = M$. If the level of cost savings is large enough so that the merger induces the exit of all outsiders, it must be the case that the total quantity is higher with the merger than without it. This follows from Lemma 1. That is, since the payoff function of an outsider firm is decreasing in σ_i, if no outsider finds it profitable to remain in the market, it must be the case that the total quantity is higher than it is without the merger. However, the rest of the results continue to hold in the case of drastic mergers. Specifically, we know from Proposition 2 that a merger can only induce exit if $c^m < \widehat{c}$. Therefore, all drastic mergers must improve consumer welfare.

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Appendix

The direct demand function derived from the utility function specified in (1) is:

$$q_i\left(p_i, \boldsymbol{p}_{-i}, N\right) = \frac{1}{\beta + \gamma (N-1)} \left[\alpha - \frac{\beta + \gamma (N-2)}{\beta - \gamma} p_i + \frac{\gamma}{\beta - \gamma} \sum_{j \neq i} p_j \right].$$

Without a merger, the equilibrium price and number of firms are determined by the first order condition

$$\frac{1}{\beta + \gamma (N^* - 1)} \left[\alpha - \frac{2\beta + \gamma (N^* - 3)}{\beta - \gamma} p^* + \frac{\beta + \gamma (N^* - 2)}{\beta - \gamma} c \right] = 0$$

and the zero profit condition

$$\pi^* (p^*, N^*; c, F) = \frac{p^* - c}{\beta + \gamma (N^* - 1)} [\alpha - p^*] - F = 0,$$

respectively.[9]

With a merger, the equilibrium number of firms and the price charged by outsiders are determined by the following post-merger first order condition of the outsider firms and the zero profit condition.

$$G(p^o, p^m, N^m) \equiv \frac{1}{\beta + \gamma (N^m - 1)} \left[\begin{array}{c} \alpha - \frac{2\beta + \gamma(N^m + M - 3)}{\beta - \gamma} p^o \\ + \frac{\beta + \gamma(N^m - 2)}{\beta - \gamma} c + \frac{\gamma}{\beta - \gamma} M p^m \end{array} \right] = 0$$

and

$$H(p^o, p^m, N^m) \equiv \frac{p^o - c}{\beta + \gamma (N^m - 1)} \left[\alpha - \frac{\beta + \gamma (M-1)}{\beta - \gamma} p^o + \frac{\gamma}{\beta - \gamma} M p^m \right] - F = 0.$$

Using these equations we obtain the following result.

Proposition 4 $N^m > (<) N^*$ *if* $p^m > (<) p^*$

Proof. First note that by the definition of p^* and N^*, if the merging firms set a price of p^* the unique equilibrium number of firms must be N^* because G and H would collapse to

$$G(p^o, p^m, N^m) \equiv \frac{1}{\beta + \gamma (N^* - 1)} \left[\alpha - \frac{2\beta + \gamma (N^* - 3)}{\beta - \gamma} p^* + \frac{\beta + \gamma (N^* - 2)}{\beta - \gamma} c \right] = 0$$

[9] Note that since $\frac{\partial^2 \pi_i (p_i, \boldsymbol{p}_{-i}, N; c, F)}{\partial p_i^2} + \sum_{j \neq i} \frac{\partial^2 \pi_i (p_i, \boldsymbol{p}_{-i}, N; c, F)}{\delta p_j \partial p_i} < 0$ and the maximized value of firm i's profit function for a given N is decreasing in N, the equilibrium is unique.

and

$$H\left(p^o, p^m, N^m\right) \equiv \frac{p^* - c}{\beta + \gamma\left(N^* - 1\right)}\left[\alpha - p^*\right] - F = 0,$$

respectively with the outsiders also setting a price of p^*.

It is therefore sufficient to show that $\frac{dN^m}{dp^m} > 0$. Using Cramer's Rule we have

$$\frac{dN^m}{dp^m} = \frac{-\frac{\partial G}{\partial p^o}\frac{\partial H}{\partial p^m} + \frac{\partial G}{\partial p^m}\frac{\partial H}{\partial p^o}}{\frac{\partial G}{\partial p^o}\frac{\partial H}{\partial N^m} - \frac{\partial G}{\partial N^m}\frac{\partial H}{\partial p^o}}$$

Evaluating these terms gives

$$\begin{aligned}
\frac{\partial G\left(p^o, p^m, N^m\right)}{\partial p^o} &= \frac{-\left(2\beta + \gamma\left(N^m + M - 3\right)\right)}{\left(\beta - \gamma\right)\left(\beta + \gamma\left(N^m - 1\right)\right)} < 0 \\
\frac{\partial G\left(p^o, p^m, N^m\right)}{\partial p^m} &= \frac{\gamma M}{\left(\beta - \gamma\right)\left(\beta + \gamma\left(N^m - 1\right)\right)} > 0 \\
\frac{\partial G\left(p^o, p^m, N^m\right)}{\partial N^m} &= -\frac{\gamma\left(p^o - c\right)}{\left(\beta - \gamma\right)\left(\beta + \gamma\left(N^m - 1\right)\right)} - \frac{\gamma}{\left(\beta + \gamma\left(N^m - 1\right)\right)^2}\left[\begin{array}{c} \alpha - \frac{2\beta + \gamma\left(N^m + M - 3\right)}{\beta - \gamma}p^o \\ + \frac{\beta + \gamma\left(N^m - 2\right)}{\beta - \gamma}c \\ + \frac{\gamma}{\beta - \gamma}Mp^m \end{array}\right] < 0 \\
\frac{\partial H\left(p^o, p^m, N^m\right)}{\partial p^m} &= \frac{\gamma\left(p^o - c\right)M}{\left(\beta - \gamma\right)\left(\beta + \gamma\left(N^m - 1\right)\right)} > 0 \\
\frac{\partial H\left(p^o, p^m, N^m\right)}{\partial p^o} &= \frac{1}{\beta + \gamma\left(N^m - 1\right)}\left[\alpha - \frac{\beta + \gamma\left(M - 1\right)}{\beta - \gamma}\left(2p^o - c\right) + \frac{\gamma}{\beta - \gamma}Mp^m\right] > 0 \\
\frac{\partial H\left(p^o, p^m, N^m\right)}{\partial N^m} &= \frac{-\gamma\left(p^o - c\right)}{\left(\beta + \gamma\left(N^m - 1\right)\right)^2}\left[\alpha - \frac{\beta + \gamma\left(M - 1\right)}{\beta - \gamma}p^o + \frac{\gamma}{\beta - \gamma}Mp^m\right] < 0.
\end{aligned}$$

Therefore, $\frac{dN^m}{dp^m} > 0$. ■