Cooperative R&D under Uncertainty with Free Entry[1]

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Abstract

This paper analyzes the effects of cooperative R&D arrangements in a model with stochastic R&D and output spillovers. Our main innovation is to allow for free entry in both the R&D race and product market. Moreover, in contrast with the literature, we assume that cooperative R&D arrangements do not have to include all the firms in the industry. We show that sharing of research outcomes is a necessary condition for the profitability of cooperative R&D arrangements with free entry. The profitability of RJV cartels depends on their size. Subsidies may be desirable in cases of larger RJVs since they are the ones which are less likely to be profitable.

JEL classification: L1, L4, O3

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1 Introduction

This paper analyzes the profit and welfare implications of cooperative R&D in an uncertain R&D environment with free entry. An important reason for the desirability of cooperative R&D arrangements is the existence of knowledge spillovers. Both input spillovers (during the R&D process) and output spillovers (after the innovation takes place) may result in inefficiently low amounts of investment. A consistent feature of the literature exploring the effects of cooperative R&D is the assumption that the number of firms participating in both the R&D process and product market is fixed.[1] In many R&D intensive industries, it is not realistic to assume that only a limited number of firms can participate in the R&D process. Profit opportunities attract entrepreneurial attention and more participants into the R&D process.[2] Moreover, the profitability of cooperative R&D arrangements may depend critically on whether there is free entry and exit in the product market, especially in case of output spillovers. Hence, our goal in this paper is to analyze the effects of cooperative R&D when the number of participants in the R&D process and product market are endogenously determined.

As in Miyagiwa and Ohno (2002), we model R&D as a stochastic process.[3] This approach differs in general from the rest of the literature where, following d'Aspremont and Jacquemin (1988), it is common to model R&D as a deterministic process.[4] Firms are assumed to participate in a game of three phases, which are denoted as the investment,

[1] See, for example, Beath et al. (1988), Combs (1992), Kamien et al. (1992), Motta (1992), Suzumara (1992), Choi (1993), Vonortas (1994), Ziss (1994), Poyago-Theotoky (1995), Leahy and Neary (1997), Katsoulacos and Ulph (1998), Salant and Shaffer (1998), Amir and Wooders (1999 and 2000), Amir (2000), Kamien and Zang (2000), Anbarci et al. (2002), Martin (2002), and Miyagiwa and Ohno (2002), and Hauenschild (2003). See De Bondt (1996) for an excellent survey.

[2] In fact, in the closely related patent race literature, it is common to assume free entry into the R&D process. See, for example, Loury (1979), Lee and Wilde (1980), Reinganum (1985), and Denicolo (2000).

[3] In many industries, the R&D process takes the form of a race. Some famous examples include the human genome race and the superconductor race. More generally, see Bresnahan and Greenstein (1999) and Burns (2006) for discussions of the high level of uncertainty that characterizes the R&D process in the computer and biotechnology industries, respectively.

[4] In addition to Miyagiwa and Ohno (2002), the other exceptions are Beath et al. (1988), Combs (1992), Choi (1993), Katsoulacos and Ulph (1998), Martin (2002), and Hauenschild (2003). Among these, only Beath et al. (1988), Martin (2002), and Miyagiwa and Ohno (2002) model the R&D process as a tournament with a single winner. Katsoulacos and Ulph (1998) differ from the rest in that they endogenize the rate of spillovers within a cooperative research arrangement.

pre-spillovers, and post-spillovers phases. In the investment stage, firms carry out research either cooperatively or non-cooperatively. We assume that cooperative R&D arrangements do not have to include all the firms in the industry. This assumption is in accordance with industrial practice, but contrasts with the previous literature where it is generally assumed that such arrangements involve all of the firms in the industry.[5] We consider an environment with output spillovers and assume that the winner of the race has exclusive access to the innovation for a limited period of time (during the pre-spillovers phase), after which the innovation spills over to all of the firms (in the post-spillovers phase). The duration of the exclusivity represents the speed of output spillovers, which could be affected by the effectiveness of patent and/or trade secret protection. Firms compete in a product market in all three phases. There is free entry and exit in both the product market and the R&D race.

Our results reveal that allowing for entry into the R&D race and product market introduces new strategic, investment and welfare implications of cooperative R&D. Following the literature, we first compare the benchmark case of R&D competition, where firms choose their R&D intensities independently, with R&D cartels, where a fixed number of firms set their investment levels to maximize their joint profits but do not share the research outcomes. We show that with free entry, such cooperative arrangements are never profitable. This result stands in stark contrast to the results in the literature, where R&D cartels are always found to be profitable.

We then consider RJV cartels, where a fixed number of firms choose their investment levels to maximize their joint profits and share their research outcomes.[6] We show that the profitability of such cooperative arrangements depends on their size. Specifically, small RJV cartels are more likely to be profitable and have higher per-firm investment levels than R&D competition while large RJV cartels are more likely to be unprofitable and have lower per-firm investment levels than R&D competition. Together with our results on R&D cartels, this implies that sharing of research outcomes is necessary for the profitability of

[5] Kamien and Zang (1993) is a notable exception.

[6] Firms do not cooperate in the product market in either of these arrangements.

cooperative R&D arrangements with free entry.

While papers which model R&D as a deterministic process always find RJV cartels to be profitable, this is not necessarily true of papers which model R&D as a stochastic process.[7] We extend the results in the stochastic R&D literature by showing that when markets are characterized by free entry, the key variable for RJV cartel performance is its size. Hence, our findings can be used to explain why RJVs often do not include all of the firms in an industry and why firms choose to conduct many R&D projects non-cooperatively.

Our analysis further reveals that the impact of cooperative R&D on the aggregate level of innovation depends on whether there are participants in the R&D race who are not part of the cooperative R&D arrangement. Interestingly, if the size of the cartel is such that some outsiders choose to participate in the race, the aggregate rate of innovation remains the same with and without a cooperative R&D arrangement, even if the number of participants in the R&D race changes. Hence, in this case, any welfare gain from R&D cooperation cannot be driven by its impact on the aggregate rate of innovation. Moreover, since R&D cartels are unprofitable, it must be the case that they are welfare-reducing. If the size of the cartel is such that no outsider firm chooses to participate in the R&D race, the aggregate rate of innovation is higher with a cooperative R&D arrangement than without one. In such cases, R&D cartels may be welfare-improving because of their positive impact on the aggregate rate of innovation and consumer welfare, and it may be desirable to subsidize them. This result is in contrast with those in the literature, where R&D cartels are generally found to be profitable, so it is never necessary to subsidize them.

Whether there are outsider participants in the R&D race or not, the impact of RJV cartels on consumer welfare is ambiguous. On the one hand, there are more firms in the pre-spillovers product market producing with the new technology under an RJV cartel. On the other hand, an RJV cartel with the new technology may cause more firms with the old technology to exit during this period. That is, the equilibrium number of firms in the pre-spillovers product market is likely to be lower with an RJV cartel. RJV cartels may be

[7] Specifically, as in our case, Beath et al. (1988), Choi (1993), and Miyagiwa and Ohno (2002) also find that RJV cartels may not always be profitable depending on the intensity of spillovers or the impact of sharing on product market payoffs.

socially desirable depending on which of these two effects dominates and there may be a case for subsiding unprofitable RJV cartels when they are welfare improving. This result is in line with the results from the literature where R&D is modelled as a stochastic process. Choi (1993) and Miyagiwa and Ohno (2002) also find room for subsidizing RJV cartels depending on the level of spillovers.[8] However, we extend their results with the surprising finding that subsidies may be desirable in case of *larger* RJVs since they are the ones which are less likely to be profitable.[9]

The paper proceeds as follows. In Section 2, we present the details of the model. Section 3 presents the product market payoffs which we use in the analysis of R&D competition, R&D cartels, and RJV cartels in Sections 4, 5, and 6, respectively. Section 7 explores the welfare and policy implications of cooperative R&D under free entry. In Section 8, we extend the analysis by considering the effects of cooperative R&D when there are no outsider participants in the R&D race in equilibrium. We conclude and make suggestions for future research in Section 9. All of the proofs are in the Appendix.

2 The Model

Consider a continuous-time model where firms participate in a game of three phases, which we denote as the investment, pre-spillovers, and post-spillovers phases. Firms compete in a product market in all three phases. In addition, firms compete to be the first to develop a new technology in the investment phase. There is free entry and exit in both the product market and the R&D race.

We assume that the product market is in a long-run equilibrium in which all participants earn zero profits when an opportunity for a new technology arises. At the beginning of the investment phase, a large (infinite) number of potential entrants decide whether or not to enter the R&D race. We allow both incumbent firms (i.e., the firms that are already active

[8] Note that this conclusion is a major departure from the results in the literature with deterministic R&D and barriers to entry, where Leahy and Neary (1997), for instance, conclude that 'policy intervention to encourage cooperation is likely to be redundant whether or not it is desirable.'

[9] As mentioned above, RJV cartel size is not a consideration in the existing models with a fixed number of firms.

in the product market when the opportunity for the new technology arises) and potential entrants to participate in the R&D race. Hence, the firms do not have to be active in the product market to be able to enter the R&D race. All participants in the R&D race incur a fixed cost S to enter the R&D race. The entry cost represents the race-specific fixed-cost expenditure.

We model the R&D race using a Poisson discovery process. Firms share a common discount rate r. Following Lee and Wilde (1980), we assume that the firms which have chosen to enter the race choose an investment x at the beginning of the race and incur a flow cost x.[10] Investment provides a stochastic time of success that is exponentially distributed with hazard rate $h(x)$. We assume that $h'(x) > 0$, $h''(x) < 0$, and $h(0) = 0$. $\lim_{x \to 0} h'(x)$ is sufficiently large to guarantee an interior equilibrium and $\lim_{x \to \infty} h'(x) = 0$.

Each firm which participates in the R&D race operates an independent research facility. However, firms may determine x, their investment level, either individually or jointly depending on whether they are part of a cooperative R&D arrangement. Following the literature, we consider three scenarios. Under R&D competition, the firms make their R&D decisions to maximize their individual payoffs. With an R&D cartel, a set $\mathbf{C} = \{1, ..., C\}$ of firms, which are exogenously designated to be part of the cartel, choose their R&D investments to maximize their joint profits. The resulting cooperative R&D agreement specifies what each firm will invest, but the cartel members do not share their research outcomes.[11] With an RJV cartel, a set $\mathbf{J} = \{1, ..., J\}$ of firms, which are exogenously designated to be part of the cartel, choose their investments to maximize their joint profits *and* all participants in the cartel acquire the new technology when and if one of the cartel's members wins the race. With an R&D or an RJV cartel, the firms cooperate only in the research stage and continue to compete in the product market.

In addition to the firms which cooperate, outsider firms may enter the R&D race if they find it profitable to do so. We let $\mathbf{R} = \{1, ..., R\}$ denote the set of all firms which choose

[10] A more general specification would allow the firms to choose an investment schedule $x(t)$ at the beginning of the race. Our formulation assumes that $x(t) = x$. Given the memoryless nature of the Poisson process we assume, this is consistent with equilibrium play in a more general set-up where each firm chooses $x(t)$.

[11] In other words, we assume that investment levels are verifiable so that the members of a cooperative R&D arrangement can write a contract on how much each member will invest.

to participate in the race. In Sections 5 and 6, we assume that some outsiders, in addition to the firms which cooperate, always find it profitable to compete in the race. That is, we focus on the part of the parameter space where $R > C$ and $R > J$ in equilibrium. We then consider in Section 8 the case where no outsiders choose to participate in the R&D race.

The investment phase ends when one of the firms develops the new technology. The pre-spillovers phase lasts for a duration of T. As stated in Miyagiwa and Ohno (2002), T can be interpreted as the speed of output spillovers. It is likely to be affected by the length and breadth of patent protection as well as the ease of reverse engineering. During this period, the winner (winners) of the R&D race has (have) exclusive rights to use the new innovation. The firms which do not have access to the new technology decide at the beginning of the pre-spillovers phase whether to participate in the product market using the old technology.

After a duration of T, the new technology becomes immediately available to all of the firms in the market (including the ones which were producing using the old technology) as well as potential entrants. In the beginning of this post-spillovers phase, each firm decides whether to participate in the product market and if it decides to do so, it chooses whether to use the new or the old technology.

As stated above, a firm does not have to participate in the R&D race to be able to participate in the product market and vice-versa. Hence, we can characterize the payoffs of the firms which are active in the product market independent of their behavior in the R&D race. It is sufficient to know the number of firms which have access to the new and old technology to determine the product market payoffs in any phase of the game. We assume that all firms incur fixed costs of production as long as they continue to produce. In other words, the fixed costs of production are an ongoing expense and not a one-off commitment. Payments on a renewable lease and head office costs are examples of these types of fixed cost.[12]

Net of fixed costs of production, let $\pi_{old}(N_{new}, N_{old})$ and $\pi_{new}(N_{new}, N_{old})$ stand for the

[12] This can be seen as a long run approximation to an industry where some costs are sunk in the short run.

flow profits of a firm which uses the old and new technology, respectively, in a market where there are N_{new} firms producing with the new technology and N_{old} firms producing with the old technology. We do not make any specific assumptions about the nature of competition that takes place in the product market. We only assume that the profit functions are decreasing in the level of competition.

Assumption 1 $\pi_i(N_{new}, N_{old})$ for $i \in \{new, old\}$ is decreasing in N_{new} and N_{old}, and is equal to zero for some $N_{new} > 0$ or $N_{old} > 0$.

Furthermore, we assume that in a product market where some firms use the new and others use the old technology, the firms which use the new technology earn higher profits.

Assumption 2 If both $N_{new} > 0$ and $N_{old} > 0$, then $\pi_{new}(N_{new}, N_{old}) > \pi_{old}(N_{new}, N_{old})$.

These two assumptions hold in standard models of Cournot and differentiated-product Bertrand competition.

We consider the symmetric subgame perfect equilibria in pure strategies of the game. To summarize, a strategy for a firm consists of (i) the decision to enter the R&D race and a choice of investment level conditional on entry, (ii) the decision to participate in the product market at the end of the R&D race, (iii) conditional on winning the race and participating in the product market, a choice of technology to use in the product market, (iv) in the post-spillovers phase, the decision to participate in the product market and a choice of technology conditional on participation.

As is standard in patent race models and in the treatment of free entry in general, we treat C, J, R, N_{old} and N_{new} as continuous variables for the purposes of differentiation and when we are solving for the zero profit number of firms.[13]

3 Product Market Competition

In this section, we start the analysis of the model by discussing the product market competition in the three phases of the game: investment, pre-spillovers, and post-spillovers.

[13] See, for example, Lee and Wilde (1980), Mankiw and Whinston (1986), and Ghosh and Morita (2007). See Seade (1980) for a justification (p. 482).

The discussion in this section is useful in determining the expected payoffs of an R&D race participant in the following sections.

In the investment phase, all firms active in the product market produce using the old technology and earn zero profits. Since firms do not have to participate in the R&D race to be able to participate in the product market and vice versa, the number of firms in the product market is determined by $\pi_{old}\left(0, N_{old}\right) = 0$.

In the pre-spillovers phase, if the innovation is not drastic, there are two types of firms in the product market, those which produce with the new technology and those which have access to the old technology only.[14] Clearly, $N_{new} = 1$ if there is R&D competition or an R&D cartel participating in the race, and N_{new} is equal to either 1 or J depending on who wins the race in an R&D environment with an RJV cartel. The number of firms using the old technology is determined endogenously by setting $\pi_{old}\left(N_{new}, N_{old}\right) = 0$. We use N_{old}^{1} and N_{old}^{J}, respectively, to denote the equilibrium number of firms which produce with the old technology when there are 1 and J firms producing with the new technology during the period T. Since $\pi_{old}\left(N_{new}, N_{old}\right) = 0$, we know from Assumption 2 that those firms which use the new technology make strictly positive profits.

If the innovation is drastic, by definition $N_{old} = 0$. We restrict attention to values of J such that all J firms find it profitable to participate in the product market. If the innovating firms expect to earn zero profits in equilibrium, they would not have any incentives to invest. Hence, such RJV cartels would not be interesting to consider.

In the post-spillovers phase, the innovation spills over to all of the participants in the product market and potential entrants. Given Assumption 2, each firm which participates in the product market chooses to use the new technology. Entry occurs until $\pi_{new}\left(N_{new}, 0\right) = 0$ and all firms earn zero profits thereafter.

[14] A drastic innovation is defined as one which causes the exit of all of the innovator's competitors. If the winner of the R&D race is an RJV cartel, multiple firms gain access to the new technology. In this case, we refer to an innovation as drastic if there are no firms participating in the product market with the old technology.

4 R&D Competition

In this section we consider the benchmark case where firms conduct R&D independently. We show that there exists a free-entry equilibrium to the R&D competition game, and characterize the investment choices and the number of firms in this equilibrium.

With a Poisson discovery process and R participants in the R&D race, the probability that there has not been a discovery until time t is given by $\exp\left[-\sum_{j\in\mathbf{R}} h(x_j)\,t\right]$.[15] During the interval $[t, t+dt)$, a generic firm i innovates with conditional probability $h(x_i)\,dt$ and earns $\frac{L}{r}$ given by

$$\int_0^T e^{-rt}\pi_{new}\left(1, N_{old}^1\right)dt = \frac{\left(1-e^{-rT}\right)\pi_{new}\left(1, N_{old}^1\right)}{r} \equiv \frac{L}{r} \tag{1}$$

in the period T before spillovers take place and zero afterwards.

With conditional probability $\sum_{j\neq i} h(x_j)\,dt$, firm i loses the race to one of its rivals during the interval dt. In this case, firm i earns zero profits both before and after spillovers take place, even if it participates in the product market in both cases.[16] This is because entry takes place until product market profits are driven to zero in both cases.

Conditional on the probability that there has not been a discovery until time t, each participant earns a flow profit of $-x_i dt$ during the interval dt whether or not they are active in the product market. This is because free entry in the product market ensures that they earn zero profits in equilibrium, as explained above. Hence, we can now write the present discounted value of the sum of firm i's expected profits over time as

$$V_i(x_i, \alpha_i) = \int_0^\infty e^{-\Sigma h(x_i)t}e^{-rt}\left[h(x_i)\frac{L}{r} - x_i\right]dt - S = \frac{h(x_i)\frac{L}{r} - x_i}{r + h(x_i) + \alpha_i} - S, \tag{2}$$

where $\alpha_i = \sum_{j\neq i} h(x_j)$ stands for the aggregate hazard rate of the rival firms.

Firm i takes α_i as given and chooses x_i to maximize (2). The first-order condition is

$$h'(x_i)\left[L + x_i + \frac{L}{r}\alpha_i\right] - \left[r + h(x_i) + \alpha_i\right] = 0. \tag{3}$$

[15] Since firms make their entry decisions at the beginning of the race, R is constant for the remainder of the race.

[16] It can participate in the product market using the old technology before spillovers take place and the new technology after spillovers take place.

The second-order condition is always satisfied because of the concavity assumption on $h(x_i)$. Hence, the first-order condition implicitly defines the best response function of firm i, $\widehat{x}(\alpha_i)$.

Since all of the firms are symmetric, we look for a symmetric equilibrium. The equilibrium per-firm investment level and number of firms can be determined by solving the first-order condition for a generic firm and the zero-profit condition simultaneously. To show that there exists a free-entry equilibrium, we need to show that the expected profits at the beginning of the race are decreasing in the number of participants, R. For a profit-maximizing firm, the envelope theorem gives

$$\frac{dV_i\left(\widehat{x}\left(\alpha_i\right),\alpha_i\right)}{dR}=\frac{\partial V_i\left(\widehat{x}\left(\alpha_i\right),\alpha_i\right)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial R} \tag{4}$$

since profit maximization implies $\frac{\partial V_i(x_i,\alpha_i)}{\partial x_i}=0$. The first term on the RHS of (4) expresses how the maximized value of (2) changes as the aggregate hazard rate of its rivals changes. Since firm i takes α_i as given and chooses x_i to maximize $V_i(x_i,\alpha_i)$, using the envelope theorem again gives

$$\frac{\partial V_i\left(\widehat{x}\left(\alpha_i\right),\alpha_i\right)}{\partial \alpha_i}=\frac{-\left[h\left(\widehat{x}\left(\alpha_i\right)\right)\frac{L}{r}-\widehat{x}\left(\alpha_i\right)\right]}{\left[r+h\left(\widehat{x}\left(\alpha_i\right)\right)+\alpha_i\right]^2}<0. \tag{5}$$

The following lemma states this result.

Lemma 1 *The expected payoff of a profit maximizing firm i that is active in the R&D race decreases monotonically with the value of α_i.*

Hence, $sign\left\{\frac{dV_i(\widehat{x}(\alpha_i),\alpha_i)}{dR}\right\}=-sign\left\{\frac{\partial\alpha_i}{\partial R}\right\}$. Note that at a symmetric equilibrium, $\alpha_i=\overline{\alpha}=(R-1)\,h(\overline{x})$, where $\overline{x}$ represents the symmetric solution to the first-order condition given in (3) taking R as given. It satisfies the following equation.

$$\overline{x}=\widehat{x}\left((R-1)\,h\left(\overline{x}\right)\right). \tag{6}$$

To determine the sign of $\frac{\partial\alpha_i}{\partial R}$, it is sufficient to determine the sign of $\frac{\partial\overline{x}}{\partial R}$. We next show that $\overline{x}$ is increasing in R. Using (6) we get

$$\frac{\partial\overline{x}}{\partial R}=\frac{\partial\widehat{x}\left((R-1)\,h\left(\overline{x}\right)\right)}{\partial R}=\frac{\frac{\partial\widehat{x}}{\partial(R-1)h(\overline{x})}h\left(\overline{x}\right)}{1-\frac{\partial\widehat{x}}{\partial(R-1)h(\overline{x})}\left(R-1\right)h'\left(\overline{x}\right)}. \tag{7}$$

Following Lee and Wilde (1980), we define the expression in the denominator as a stability condition, which implies that it is positive. The expression in the numerator is also positive because the investment decisions of the firms are strategic complements. To see this, note that using the implicit function theorem gives us

$$\frac{\partial \widehat{x}\left(\alpha_i\right)}{\partial x_j} = -\frac{h'\left(x_j\right)\left[h'\left(\widehat{x}\right)\frac{L}{r} - 1\right]}{h''\left(\widehat{x}\right)\left[L + \widehat{x} + \frac{L}{r}\alpha_i\right]}. \tag{8}$$

The first-order condition given in (3) can be re-written as

$$\left[h'\left(x_i\right)\frac{L}{r} - 1\right]\left[r + h\left(x_i\right) + \alpha_i\right] - h'\left(x_i\right)\left[h\left(x_i\right)\frac{L}{r} - x_i\right] = 0. \tag{9}$$

Substituting in (2) and rearranging give us $V_i + S = \frac{h(x_i)\frac{L}{r} - x_i}{r + h(x_i) + \alpha_i} = \frac{\left[h'(x_i)\frac{L}{r} - 1\right]}{h'(x_i)}$. Hence, we can re-write (8) as

$$\frac{\partial \widehat{x}\left(\alpha_i\right)}{\partial x_j} = -\frac{h'\left(x_j\right)h'\left(\widehat{x}\right)\left(V_i + S\right)}{h''\left(\widehat{x}\right)\left[L + \widehat{x} + \frac{L}{r}\alpha_i\right]} \tag{10}$$

and note that, because $h''\left(x_i\right) < 0$, firms' investment choices are strategic complements for a given number of R&D race participants (R). This contrasts with the results in Miyagiwa and Ohno (2002), where firms' investments are only strategic complements for sufficiently fast spillovers. The reason for this difference is that in our case, because of free entry in the product market, the firms which lose the R&D race do not benefit from spillovers.

We can now conclude that $\frac{\partial \overline{x}}{\partial R} > 0$ and hence $\frac{\partial \overline{\alpha}}{\partial R} > 0$. Defining the corresponding per-firm profit level for a given value of R as

$$\overline{V}\left(R\right) = \frac{h\left(\overline{x}\right)\frac{L}{r} - \overline{x}}{r + Rh\left(\overline{x}\right)} - S, \tag{11}$$

we have the following result.

Proposition 1 *There exists a free-entry equilibrium to the R&D competition game where the equilibrium number of firms, R^N, is determined by $V^N = \overline{V}\left(R^N\right) = 0$. Each of these R^N firms invests $x^N = \overline{x}\left(R^N\right)$.*

5 R&D Cartel

In this section, we compare the case of R&D competition with the case where a group of C firms form an R&D cartel. The cartel members participate in the R&D race by choosing their investment levels to maximize their joint payoffs, but they do not share their research outcomes. Due to free entry in the R&D race, the cartel participants still face competition from outsider participants in the race.

The joint payoffs of the cartel participants are given by

$$\sum_{i\in\mathbf{C}}\left(\frac{h(x_i)\frac{L}{r}-x_i}{r+\sum\limits_{j\in\mathbf{C}}h(x_j)+\sum\limits_{k\notin\mathbf{C}}h(x_k)}-S\right), \tag{12}$$

where $\mathbf{C}$ is the set of firms participating in the R&D cartel, $\frac{L}{r}$ is as defined in (1), and the last term in the denominator, $\sum\limits_{k\notin\mathbf{C}} h(x_k)$, stands for the sum of the hazard rates of the outsider participants in the race. Each outsider maximizes the payoff function given in (2).

We look for a symmetric equilibrium, where each cartel member invests x^C, each outsider participant invests x^O, and R^C stands for the number of participants in the R&D race. These values, if an equilibrium exists, can be found by solving the first-order conditions given by

$$h'\left(x^C\right)\left[L+Cx^C+\frac{L}{r}\left(R^C-C\right)h\left(x^O\right)\right]-\left[r+Ch\left(x^C\right)+\left(R^C-C\right)h\left(x^O\right)\right]=0 \tag{13}$$

and

$$h'\left(x^O\right)\left[L+x^O+\frac{L}{r}\left(\begin{array}{c}\left(R^C-C-1\right)h\left(x^O\right)\\+Ch\left(x^C\right)\end{array}\right)\right]-\left[\begin{array}{c}r+Ch\left(x^C\right)\\+\left(R^C-C\right)h\left(x^O\right)\end{array}\right]=0, \tag{14}$$

and the zero-profit condition given by

$$\frac{h\left(x^O\right)\frac{L}{r}-x^O}{r+Ch\left(x^C\right)+\left(R^C-C\right)h\left(x^O\right)}-S=0 \tag{15}$$

simultaneously.[17]

The following proposition establishes that there exists a free-entry equilibrium with an R&D cartel. As in the case of R&D competition, the result relies on a stability condition specified in the Appendix.

Proposition 2 *There exists a free-entry equilibrium with an R&D cartel.*

[17] It is straightforward to verify that the second-order conditions hold because of the concavity assumption on $h(x_i)$.

To determine the profitability of R&D cartels and their impact on innovation, we start by comparing the per-firm investment level under R&D competition with the per-firm investment level in a race with an R&D cartel. The following proposition establishes that while the cartel participants reduce their per-firm investment level, the outsider participants invest the same as they do under R&D competition.

Proposition 3 *In an R&D race with an R&D cartel, the equilibrium per-firm investment level of the R&D cartel participants, x^C, is lower than the equilibrium per-firm investment level under R&D competition, x^N. The equilibrium investment level of the outsider firms, x^O, is equal to x^N.*

With free entry in the product market, output spillovers do not provide any benefit to the firms that lose the R&D race. This transforms the race into a winner-takes-all game, where R&D investments always confer a net negative externality on rivals by decreasing their chances of winning the race. Since the cartel members internalize the negative externality they impose on each other, they invest less than the outsider participants in the race.

The outsider firms invest the same amount in the presence of an R&D cartel as they do under R&D competition. This somewhat surprising result follows from our assumption of free entry in the R&D race. With a fixed number of participants in the race, the formation of an R&D cartel would cause each outsider firm's expected payoff to increase and investment level to decrease because the cartel members invest less than they do under R&D competition.[18] With free entry into the race, the increase in the expected profits of the outsider firms invites entry into the race until the expected profits are driven down to zero. Since an outsider firm i earns zero profits both with and without an R&D cartel in the race, Lemma 1 implies that it must face the same value of α_i, solve the same profit maximization

[18] It is straightforward to check that the outsider firms' reaction function is upward sloping. For given values of R^C and C, we get from (14) that

$$\frac{dx^O}{dx^C} = -\frac{Ch'\left(x^C\right)\left[h'\left(x^O\right)\frac{L}{r}-1\right]}{h''\left(x^O\right)\left[L+x^O+\frac{L}{r}\left(\left(R^C-C-1\right)h\left(x^O\right)+Ch\left(x^C\right)\right)\right]+\left(R^C-C-1\right)h'\left(x^O\right)\left[h'\left(x^O\right)\frac{L}{r}-1\right]},$$

which is positive because the denominator is negative due to the stability condition specified in Section 1 of the Appendix.

problem, and invest the same amount in both cases.[19]

Comparing these results with those of Miyagiwa and Ohno (2002) reveals the importance of the assumption of free entry. If there are barriers to entry in the product market, the losers of the R&D race still get a chance to benefit from the innovation after spillovers happen. For sufficiently small values of T (i.e., for sufficiently rapid spillovers), this positive spillover effect outweighs the negative competitive effect mentioned above and, thus, R&D investments confer a net positive externality on the rival firms. Hence, Miyagiwa and Ohno (2002) find that for sufficiently small T values, members of an R&D cartel internalize these positive externalities and increase their investment levels above the investment level under R&D competition.[20]

Ultimately, what is important is the impact of the R&D cartel on the aggregate arrival rate of innovation. The conclusion in the literature is that R&D cartels decrease the aggregate rate of innovation for sufficiently low spillovers and increase it for sufficiently high spillovers. This follows immediately from the per-firm investment results discussed above since it is generally assumed that all firms participate in the R&D cartel. In our context, due to the assumption of free entry, one cannot readily use the results on the individual investment levels to determine the impact of R&D cartels on the aggregate rate of innovation. We have the following result.

Proposition 4 *In an R&D environment with an R&D cartel,*

(i) the aggregate arrival rate of innovation is the same as under R&D competition;

(ii) a higher number of firms participate in the R&D race than under R&D competition.

Although the cartel members invest less than they would under R&D competition, the aggregate rate of innovation does not decrease with the formation of an R&D cartel. This is because the decrease in the investment levels of the cartel members makes entry more

[19] A numerical example, which illustrates the results stated in Proposition 3 in a set-up with Cournot oligopoly, linear demand and constant marginal cost, is available from the authors upon request.

[20] Miyagiwa and Ohno's (2002) result is in line with the results of the other papers in the literature which analyze R&D cartels in a model with a deterministic R&D process. See, for example, d'Aspremont and Jacquemin (1988) and Kamien et al. (1992).

attractive. Proposition 4 implies that the entrants' investment level exactly compensates for the decrease in the investment level of the R&D cartel members.

We finally evaluate the profitability of R&D cartels.

Proposition 5 *All R&D cartels are unprofitable.*

This result also contrasts with the results in the previous studies of R&D cartels, which consistently find that the joint profits of the firms within an R&D cartel are higher than their joint profits under R&D competition.[21] In our analysis, free entry into the race makes otherwise profitable R&D cartels unprofitable. This is because the members of an R&D cartel earn less than the outsider participants in the race because of a free rider effect.[22] The outsider firms benefit from the lower investment of the cartel members because it increases their probability of success. Since outsiders earn zero and cartel members earn less, R&D cartels are unprofitable in equilibrium.[23]

6 RJV Cartel

We next consider the case where an exogenously-determined group of J firms form an RJV cartel. The firms make their investment decisions jointly and gain immediate access to the new technology if any one of them wins the race.

The RJV cartel's expected payoff is

$$\sum_{i \in \mathbf{J}} \left(\frac{h(x_i)\frac{L^J}{r} + \sum\limits_{k \neq i, k \in \mathbf{J}} h(x_k)\frac{L^J}{r} - x_i}{r + h(x_i) + \sum\limits_{k \neq i, k \in \mathbf{J}} h(x_k) + \sum\limits_{l \notin \mathbf{J}} h(x_l)} - S \right), \tag{16}$$

where the last term in the denominator stands for the sum of the hazard rates of the outsider participants in the race. $\frac{L^J}{r}$ represents the present discounted value of the profit that each

[21] See, for example, d'Aspremont and Jacquemin (1988), Kamien et al. (1992), and Miyagiwa and Ohno (2002).

[22] A similar kind of free rider effect exists in the mergers literature. See Salant et al. (1983) and Deneckere and Davidson (1985).

[23] Davidson and Mukherjee (2007) and Erkal and Piccinin (2007) show that free entry in the product market makes otherwise profitable product market mergers unprofitable.

member of the RJV cartel expects to make if the winner of the race is one of the cartel members. It is given by

$$\frac{L^J}{r} = \frac{\left(1 - e^{-rT}\right) \pi_{new}\left(J, N_{old}^J\right)}{r}, \tag{17}$$

where N_{old}^J is the equilibrium number of firms producing with the old technology during the period T. The flow profit the firms earn if they win the race, $\pi_{new}\left(J, N_{old}^J\right)$, depends on the number of participants in the RJV cartel because the winner shares the new technology with the rest of the cartel, which determines the number of firms in the product market with the new technology for the period T. In the following analysis, we explore how the performance of RJV cartels changes as L^J changes. In contrast with the case of R&D cartels, we will see that RJV cartel members may invest more or less than they would under R&D competition depending on the value of L^J.

We start the analysis by establishing that there exists a free-entry equilibrium with an RJV cartel where each cartel member invests x^J, each outsider participant invests x^O, and R^J stands for the number of participants in the R&D race. In equilibrium, these values must satisfy the first-order conditions given by

$$Jh'\left(x^J\right)\left[L^J + x^J + \frac{L^J}{r}\left(R^J - J\right) h\left(x^O\right)\right] - \left[r + Jh\left(x^J\right) + \left(R^J - J\right) h\left(x^O\right)\right] = 0 \tag{18}$$

and

$$h'\left(x^O\right)\left[L + x^O + \frac{L}{r}\left[\begin{array}{c}\left(R^J - J - 1\right) h\left(x^O\right) \\ +Jh\left(x^J\right)\end{array}\right]\right] - \left[\begin{array}{c} r + Jh\left(x^J\right) \\ +\left(R^J - J\right) h\left(x^O\right)\end{array}\right] = 0, \tag{19}$$

and the zero-profit condition given by

$$\frac{h\left(x^O\right) \frac{L}{r} - x^O}{r + Jh\left(x^J\right) + \left(R^J - J\right) h\left(x^O\right)} - S = 0.^{24} \tag{20}$$

As in the case of R&D competition and R&D cartels, the result relies on a stability condition specified in the Appendix.

Proposition 6 *There exists a free-entry equilibrium with an RJV cartel.*

[24] It is straightforward to verify that the second-order conditions hold because of the concavity assumption on $h\left(x_i\right)$.

The difference between an RJV cartel and an R&D cartel is in the product market payoffs the members receive when one of the cartel participants wins the race. Hence, to determine the profitability of RJV cartels and their impact on innovation, we first explore how the equilibrium per-firm investment and profit level of an RJV cartel member change with L^J. Since L^J stands for the benefit from winning the race for each member of the RJV cartel, its magnitude for a given value of J would depend on the nature and intensity of competition in the product market. While analyzing the impact of a change in L^J, one also has to take into account its effect on the entry and investment decisions of the outsider participants in the R&D race. We have the following result.

Lemma 2 *The equilibrium per-firm investment and profit levels of an RJV cartel of size J are monotonically increasing in L^J.*

We next evaluate the performance of an RJV cartel for low and high values of L^J to draw conclusions for the range of possible RJV cartel effects. The following lemma presents results for the cases when $L^J = \frac{L}{J}$ and $L^J = L$. In the first case, industry profits remain unaffected by the formation of the RJV cartel. Since more firms have access to the new technology with an RJV cartel, the cartel members face higher competition in the product market and divide among themselves what they would have earned on their own under R&D competition. In the second case, each cartel member's profit in the market is equal to what it would have earned under R&D competition. This case may arise if the existence of several firms with access to the new technology causes many firms using the old technology to exit the market. As a result, the RJV cartel members end up facing less competition overall even if there are more firms which have access to the new technology with an RJV cartel. The RJV cartel clearly increases industry profits in this case.

Lemma 3 *If $L^J = \frac{L}{J}$, in equilibrium the members of an RJV cartel invest less than x^N, the per-firm investment level under R&D competition, and make a lower profit than they would under R&D competition. If $L^J = L$, in equilibrium the members of an RJV cartel invest more than x^N and make a higher profit than they would under R&D competition.*

In the analysis of the impact of an RJV cartel on the per-firm investment level, two effects play a role. First, under R&D competition, one firm's investment decreases the expected profits of another firm because it reduces the probability that the second firm will win the race. However, if research outcomes are shared, one firm's investment increases another firm's expected profits because it increases the probability that the second firm will have access to the new technology immediately after the race. Joint profit maximization within an RJV cartel allows the cartel participants to internalize these positive externalities, which causes the per-firm investment to increase. Second, the per-firm returns to winning when the firms are part of an RJV cartel differ from those under R&D competition because when a member of the RJV cartel wins the R&D race, all of its members have access to the new technology. When $L^J = \frac{L}{J}$, the per-firm returns with an RJV cartel are lower than those under R&D competition, which are equal to L. This affects the per-firm investment level with an RJV cartel adversely. Lemma 3 implies that when $L^J = \frac{L}{J}$, this negative effect dominates the positive effect and, hence, the members of the RJV cartel invest less than they would under R&D competition and make lower profits.[25] On the other hand, when $L^J = L$, the returns to winning are the same under both arrangements, and the first effect causes the per-firm investment and profit level to be higher with an RJV cartel.

In the proof of Lemma 3, we also show that when $L^J = \frac{L}{J}$, an RJV cartel and an R&D cartel of the same size result in the same per-firm investment and profit level in equilibrium. Together with Lemma 2, this implies that for $L^J > \frac{L}{J}$, firms make higher investments and profits in an RJV cartel than in an R&D cartel. The outsider firms, on the other hand, invest x^N in both cases, irrespective of the value of L^J.

Lemmas 2 and 3 imply the existence of two critical values, $\widetilde{L}^J(J)$ and $\widehat{L}^J(J)$, such that the per-firm investment and profit level are higher with an RJV cartel than under R&D competition if $L^J > \widetilde{L}^J(J)$ and $L^J > \widehat{L}^J(J)$, respectively. In the following proposition, we establish that $\widehat{L}^J(J) < \widetilde{L}^J(J)$ and characterize the performance of RJV cartels based on

[25] This result implies that RJV cartels may be unprofitable even in those cases when sharing increases industry profits because of free entry in the R&D race. In contrast, Miyagiwa and Ohno (2002) find that if industry profits increase with the sharing a new technology, members of an RJV cartel must be making higher profits than they would under R&D competition.

L^J.

Proposition 7 *For values of J such that $L^J > \widetilde{L}^J(J) \in \left(\widehat{L}^J(J), L\right)$, members of an RJV cartel invest higher amounts and make higher profits than they would under R&D competition. For values of J such that $L^J \in \left[\widehat{L}^J(J), \widetilde{L}^J(J)\right]$, members of an RJV cartel invest lower amounts and make higher profits than they would under R&D competition. For values of J such that $L^J < \widehat{L}^J(J) \in \left(\frac{L}{J}, L\right)$, members of an RJV cartel invest lower amounts and make lower profits than they would under R&D competition.*

Proposition 7 implies that if the per-firm investment level is higher with an RJV cartel, the per-firm profit level must also be higher. The reason that the firms may earn higher profits than they would under R&D competition even if their investment levels are lower is that being a member of an RJV cartel provides them with insurance. They start to earn L^J as soon as any member of the cartel wins the race. Hence, with lower individual investments, they can still have higher individual expected payoffs than they would under R&D competition.

Proposition 7 allows us to link RJV cartel size to RJV cartel performance if we impose a weak condition on the relationship between L^J and J.

Assumption 3 $\frac{dL^J\left(J,N_{old}^J\right)}{dJ} = \frac{\partial L^J\left(J,N_{old}^J\right)}{\partial J} + \frac{\partial L^J\left(J,N_{old}^J\right)}{\partial N_{old}} \cdot \frac{\partial N_{old}^J}{\partial J} \leqslant 0.$

Assumption 3 states that the returns to an individual firm from winning the R&D race are weakly decreasing in the size of the RJV cartel, even after taking into account the fact that as the number of firms in the RJV cartel increases, the number of post-innovation competitors the firm faces decreases. This assumption is satisfied in many standard models of oligopolistic competition with free entry.[26]

Given Assumption 3, the following corollary follows immediately from Proposition 7.

Corollary 1 *Members of a sufficiently small RJV cartel such that $L^J > \widetilde{L}^J(J)$ invest higher amounts and make higher profits than they would under R&D competition. Members*

[26] It is straightforward to verify that Assumption 3 is satisfied in a model with linear demand and Cournot competition. Additional examples can be given using models of logit and CES demand systems.

of an intermediate-sized RJV cartel such that $L^J \in \left[\widehat{L}^J(J), \widetilde{L}^J(J)\right]$ *invest lower amounts and make higher profits than they would under R&D competition. Members of a sufficiently large RJV cartel such that* $L^J < \widehat{L}^J(J)$ *invest lower amounts and make lower profits than they would under R&D competition.*

Corollary 1 compares RJV cartels of various size categories with the benchmark of R&D competition. The per-firm payoff to winning the R&D race, L^J, depends on the RJV cartel's size, J. For sufficiently large RJV cartels such that $L^J < \widehat{L}^J(J)$, these payoffs are lower because the innovation is shared amongst more firms in the product market, which increases product market competition. Indeed, some large RJV cartels may be unprofitable because they include too many firms. Hence, Corollary 1 provides an explanation for why relatively small RJV cartels may form - these may be more profitable than larger ones.

While papers that model R&D as a deterministic process always find RJV cartels to be profitable, this is not necessarily true of papers that model R&D as a stochastic process.[27] We extend the results in the stochastic R&D literature by showing that when markets are characterized by free entry, the key variable for RJV cartel performance is its size. Our findings can be used to explain why RJVs often do not include all of the firms in an industry and why firms choose to conduct many R&D projects non-cooperatively. This analysis makes an important contribution to the literature since studies of cooperative arrangements in R&D environments generally assume that all of the firms in the industry participate in the cooperative structure. An exception is Kamien and Zang (1993). Using a model with barriers to entry and a deterministic R&D process, they find that if the firms in an industry form competing RJV cartels, the resulting aggregate investment level may be higher than if all of the firms were members of the same grand RJV cartel. However, in Kamien and Zang's (1993) model, firms always earn higher profits with a grand RJV than with competing RJVs because as the size of the cartel increases, the cartel members face less competition during the R&D process. In contrast, having a larger RJV does not

[27] Specifically, as in our case, Beath et al. (1988), Choi (1993), and Miyagiwa and Ohno (2002) also find that RJV cartels may not always be profitable depending on the intensity of spillovers or the impact of sharing on product market payoffs.

necessarily result in less competition during the R&D process in our case due to free entry by outsider firms. Smaller RJVs may be more profitable because although the benefits from joint profit maximization are lower with a smaller RJV, each firm expects to earn more in the product market.

Finally, we turn our attention to the impact of RJV cartels on the aggregate arrival rate of innovation. Since the analysis is identical to the analysis in the case of R&D cartels, we do not repeat it here. We get the following result.

Proposition 8 *The aggregate arrival rate of innovation with an RJV cartel is the same as under R&D competition.*

This result differs from the results in the literature with barriers to entry. In the deterministic R&D literature, RJV cartels always increase the aggregate arrival rate of innovation. In contrast, in the stochastic R&D literature, the impact of RJV cartels on the aggregate arrival rate of innovation critically depends on the degree and type of spillovers.[28] Our analysis extends these results by pointing out that with free entry, even if the cartel members invest different amounts than they do under R&D competition, the aggregate arrival rate of innovation remains the same under the two scenarios.

Combined with Proposition 4, this result implies that the aggregate rate of innovation is independent of whether the firms form an R&D cartel or an RJV cartel. This is so despite the fact that the number of participants in the R&D race varies with the type of cooperation. To see this, note that for an R&D and an RJV cartel of the same size (i.e., assuming $J = C$), it is straightforward to show, using the payoff functions (12) and (16), that $x^C \gtreqless x^J$ depending on $L^J \lesseqgtr \frac{L}{J}$. This implies that it must be the case that $R^J \gtreqless R^C$ depending on $L^J \lesseqgtr \frac{L}{J}$ since the aggregate rate of innovation is the same under the two regimes and the outsider firms invest x^N in both cases. Propositions 4 and 8 further imply that since $x^C < x^N$, it must be the case that $R^C > R^N$, and since $x^J \gtreqless x^N$ depending

[28] For example, Beath et al. (1988) and Choi (1993) show that aggregate R&D increases with the formation of an RJV cartel if the rate of spillovers is sufficiently high. Miyagiwa and Ohno (2002) show that the impact of RJV cartels on the aggregate arrival rate of innovation depends on the level of spillovers as well as the effect of sharing on industry profits. Hauenschild (2002) shows that input spillovers increase R&D investments while output spillovers decrease R&D investments.

on $L^J \gtreqless \widetilde{L}^J(J)$, it must be the case that $R^J \lesseqgtr R^N$ depending on $L^J \gtreqless \widetilde{L}^J(J)$. That is, although the number of R&D race participants is higher with an R&D cartel than under R&D competition, in the case of RJV cartels, it is the sufficiently large ones that result in a higher number of R&D race participants than under R&D competition.

7 Welfare and Policy Implications

We next turn our attention to the welfare and policy implications of cooperative R&D. We define welfare as the sum of consumer welfare and producer surplus. This implies that since firms earn zero profits in equilibrium, welfare under R&D competition is equal to

$$W^N = \frac{R^N h\left(x^N\right) \frac{\Omega^1}{r} + \omega^0}{r + R^N h\left(x^N\right)}, \tag{21}$$

where

$$\frac{\Omega^1}{r} = \frac{\left(1 - e^{-rT}\right)\omega^1 + e^{-rT}\omega^{all}}{r} \tag{22}$$

stands for the consumer welfare level after the race ends. The superscript 1 denotes the case when only one firm has access to the new technology for duration T. We use ω^0, ω^1 and ω^{all} to denote the *flow* consumer welfare when no firms, only one firm, and all firms have access to the new technology, respectively.

Similarly, the equilibrium welfare expressions with an R&D and RJV cartel are

$$W^C = \frac{\left[Ch\left(x^C\right) + \left(R^C - C\right)h\left(x^N\right)\right]\frac{\Omega^1}{r} + \omega^0 + C\left[h\left(x^C\right)\frac{L}{r} - x^C\right]}{r + Ch\left(x^C\right) + \left(R^C - C\right)h\left(x^N\right)} - C \cdot S \tag{23}$$

and

$$W^J = \frac{Jh\left(x^J\right)\frac{\Omega^J}{r} + \left(R^J - J\right)h\left(x^N\right)\frac{\Omega^1}{r} + \omega^0 + J\left[h\left(x^J\right)\frac{JL^J}{r} - x^J\right]}{r + Jh\left(x^J\right) + \left(R^J - J\right)h\left(x^N\right)} - J \cdot S, \tag{24}$$

respectively. In (24), defining ω^J as the flow consumer welfare when J firms have the new technology,

$$\frac{\Omega^J}{r} = \frac{\left(1 - e^{-rT}\right)\omega^J + e^{-rT}\omega^{all}}{r} \tag{25}$$

stands for the consumer welfare level after the race ends.

Since we evaluate welfare from an ex ante perspective, the aggregate arrival rate of innovation determines how rapidly consumers start to benefit from the new technology and firms start to make profits from it. Although Propositions 4 and 8 state that the aggregate rate of innovation remains unchanged with R&D and RJV cartels, we show in the following discussion that their formation may still affect welfare.

7.1 R&D Cartels

Since the innovation arrives at the same time in expectation whether or not there is an R&D cartel, we have

$$W^C - W^N = \frac{C\left[h\left(x^C\right)\frac{L}{r} - x^C\right]}{r + Ch\left(x^C\right) + \left(R^C - C\right)h\left(x^N\right)} - C \cdot S. \tag{26}$$

That is, the only difference between the welfare level with an R&D cartel and the welfare level under R&D competition is the expected profits of the R&D cartel members themselves. Consumer welfare is the same in expectation whether or not an R&D cartel is formed because in both cases there is only one firm with the new technology in the market for the duration T after the R&D race ends.

Since we know from Proposition 5 that R&D cartels earn negative profits, (26) implies that they must be welfare decreasing. Hence, our analysis implies that in industries with free entry, R&D cartels would never arise and antitrust policy towards them is irrelevant. Moreover, since they always decrease welfare, it is not desirable to subsidize R&D cartels in order to make them profitable. We show in Section 8 that this conclusion may change if there are no outsiders choosing to participate in the R&D race.

7.2 RJV Cartels

As in the case of R&D cartels, since $Jh\left(x^J\right) + \left(R^J - J\right)h\left(x^N\right) = R^N h\left(x^N\right)$, we have

$$W^J - W^N = \frac{Jh\left(x^J\right)\frac{\Omega^J}{r} + \left(R^J - J - R^N\right)h\left(x^N\right)\frac{\Omega^1}{r}}{r + Jh\left(x^J\right) + \left(R^J - J\right)h\left(x^N\right)} + J\left[\frac{h\left(x^J\right)\frac{JL^J}{r} - x^J}{r + Jh\left(x^J\right) + \left(R^J - J\right)h\left(x^N\right)} - S\right]. \tag{27}$$

That is, the difference between consumer welfare with an RJV cartel and under R&D competition is that when an RJV cartel wins the race but before spillovers occur, there are

J firms with the new technology rather than only one. Hence, we can conclude from (22) and (25) that any profitable RJV cartel is also welfare improving if $\omega^J \geqslant \omega^1$. While one would expect this inequality to hold (i.e., consumer welfare to be increasing in N_{new}) in a market structure where the total number of firms is exogenously determined, this may not always be the case in a market where there is free entry and exit of firms with access to the old technology (i.e., N_{old} is endogenously determined). This is because increasing the number of firms with access to the new technology may cause greater exit of firms with the old technology. Therefore, there may be fewer firms (in total) active in the product market when there are more firms with access to the new technology. In general, we cannot conclude whether the potential negative effects of having a smaller total number of firms in the market (which results in less variety or potentially less competition) is completely offset by having more firms in the market with access to the new technology. For this reason, it is possible for consumer welfare to be lower when an RJV cartel wins the race than when a single firm does. As a result, profitable RJV cartels may present a welfare trade-off between lower expected consumer welfare and higher expected profits.

The analysis also implies that there may be a case for subsiding unprofitable RJV cartels when they are welfare improving. This result is in line with the results from the literature with stochastic R&D, where Choi (1993) and Miyagiwa and Ohno (2002) also find room for subsidizing RJV cartels. In a tournament model like ours, Miyagiwa and Ohno (2002) conclude that it is both privately and socially optimal to form an RJV cartel if spillovers are fast and industry profits from sharing exceed those without sharing.[29] Otherwise, "there is no guarantee that the R&D regime that the industry selects is the best for society" (p. 868). Hence, they do not identify when, if at all, government support would be desirable. Our analysis takes us a step closer to this, with the surprising result that subsidies may be desirable in case of larger RJVs.[30] In Choi's (1993) non-tournament framework, social

[29] Note that in our model, these two conditions together are neither necessary nor sufficient for the social and private incentives for RJV cartels to coincide because of the product market exit induced by successful RJV cartels.

[30] It may, however, be the case that where large unprofitable RJV cartels are welfare improving, smaller profitable RJV cartels are also welfare improving, and perhaps even more so. To formulate policy in this area, a careful study of endogenous RJV formation would be necessary, together with a comparison of the

incentives to form RJV cartels always exceed private incentives. However, Choi's (1993) results depend upon the assumption that sharing research outcomes increases product market competition. Such an assumption is not necessary for our conclusions, which are driven by the extra pressure put on members of cooperative arrangements by entrants.[31]

8 Cooperative R&D without R&D race outsiders

In the analysis so far, we have maintained the assumption that some outsiders always find it profitable to enter the R&D race in equilibrium. In this section, we provide some additional insights about cooperative R&D with free entry for the case where no outsiders choose to enter the race. We do this to address the potential concern that cooperation between firms in the R&D race may induce the exit of outsiders and, thus, reduce competition in the R&D race. Our results in this section show that the prospect of cooperative R&D having this effect is no cause for concern.

It is straightforward to show that most of the results from the previous analysis continue to hold in a set-up without R&D race outsiders.[32] The main difference is in the result concerning the aggregate rate of innovation. Surprisingly, we show in the following proposition that without outsiders, the aggregate rate of innovation with either an R&D or an RJV cartel must be at least as high as it is under R&D competition. This is because if outsiders find it unprofitable to enter the race, it must be because the cooperating firms have collectively invested enough to ensure that any entry would be unprofitable.

Proposition 9 *If there are no outsiders in equilibrium, the aggregate rate of innovation with an R&D cartel or an RJV cartel must be at least as high as it is under R&D competition.*

As a result of Proposition 9, the only policy conclusion that is qualitatively different from the ones we reached in Section 7 is that it may be desirable to subsidize those R&D

welfare implications of RJV cartels of different sizes. This issue is beyond the scope of this paper and would be interesting to pursue as future research.

[31] These results contrast with the results in the literature with deterministic R&D and barriers to entry, where policy intervention to encourage cooperation is never desirable.

[32] The results are available from the authors upon request. In particular, we show that all R&D cartels are unprofitable and their per-firm investment is less than x^N. Moreover, there are critical values of L^J above which RJV cartels invest more per-firm than x^N and are profitable.

cartels which increase the aggregate rate of innovation since they increase consumer welfare in expectation. Hence, R&D cartels without R&D race outsiders may present a welfare trade-off between lower profits and higher consumer welfare. To the best of our knowledge, the conclusion that subsidies for R&D cartels may be socially desirable is unique in the literature since they are always found to be profitable.

9 Conclusion

We have analyzed the effects of cooperative R&D in a model of free entry with a stochastic R&D process and oligopolistic product market. Our findings account for the effects of entry and exit in R&D environments which have been missing from the literature to date. In contrast with the results in the literature, we have shown that R&D cartels are always unprofitable and never affect the aggregate rate of innovation adversely in equilibrium. RJV cartels, on the other hand, can be profitable depending on their size. Similar to R&D cartels, they also never adversely affect the aggregate rate of innovation.

Both the standard approach of modelling cooperative R&D with barriers to entry and our approach of free entry can be understood as opposite ends of a spectrum. This paper offers some guidance as to how the existing literature's policy prescriptions may change as entry conditions vary along this continuum. The concern that competing firms in most R&D environments may have too little incentives to invest in R&D due to spillovers and appropriability problems has caused both the US and Europe to pass legislation for lenient antitrust treatment of research joint ventures (RJVs).[33] Our results indicate that it may be desirable to subsidize R&D cartels in cases when there are no outsider participants in the R&D race. Such a policy conclusion does not find support in the existing literature which assumes barriers to entry because a consistent conclusion of this literature is that R&D cartels are always profitable. The results also imply that since sharing of R&D outcomes

[33] In the US, the National Cooperative Research and Production Act (NCRPA) of 1993 provides that research and production joint ventures be subject to a 'rule of reason' analysis instead of a per se prohibition in antitrust litigation. In the EU, the Commission Regulation (EC) No 2659/2000 (the EU Regulation) provides for a block exemption from antitrust laws for RJVs, provided that they satisfy certain market share restrictions and allow all joint venture participants to access the outcomes of the research.

affects the equilibrium number of firms in the product market after the R&D race, it is possible for consumer welfare to be lower when an RJV cartel wins the race than when a single firm does. Hence, the optimal antitrust treatment of cooperative R&D arrangements may be different for different industries and a detailed analysis of demand may be required to determine the appropriate policy approach. Subsidies may be desirable in cases of larger RJVs since they are the ones which are less likely to be profitable.

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Appendix

1 Proof of Proposition 2

Let $\overline{x}^C$ and $\overline{x}^O$ stand for the investment levels which satisfy the first-order conditions of the cartel participants and the outsider firms, respectively, for a given number of cartel participants, C, and outsiders, $O = R - C$. We first show that, as in the case of R&D competition, $\overline{x}^C$ and $\overline{x}^O$ are both increasing in R by invoking a stability condition.

For given values of C and R, let $\overline{G}^C$ and $\overline{H}^C$ represent the first-order conditions given in (13) and (14). $\overline{G}^C$ and $\overline{H}^C$ implicitly define $\overline{x}^C$ and $\overline{x}^O$. Totally differentiating and applying Cramer's Rule gives

$$\frac{d\overline{x}^C}{dR} = \frac{-\frac{\partial \overline{G}^C}{\partial R}\frac{\partial \overline{H}^C}{\partial \overline{x}^O} + \frac{\partial \overline{G}^C}{\partial \overline{x}^O}\frac{\partial \overline{H}^C}{\partial R}}{\frac{\partial \overline{G}^C}{\partial \overline{x}^C}\frac{\partial \overline{H}^C}{\partial \overline{x}^O} - \frac{\partial \overline{G}^C}{\partial \overline{x}^O}\frac{\partial \overline{H}^C}{\partial \overline{x}^C}} \tag{A.1}$$

and

$$\frac{d\overline{x}^O}{dR} = \frac{-\frac{\partial \overline{G}^C}{\partial \overline{x}^C}\frac{\partial \overline{H}^C}{\partial R} + \frac{\partial \overline{G}^C}{\partial R}\frac{\partial \overline{H}^C}{\partial \overline{x}^C}}{\frac{\partial \overline{G}^C}{\partial \overline{x}^C}\frac{\partial \overline{H}^C}{\partial \overline{x}^O} - \frac{\partial \overline{G}^C}{\partial \overline{x}^O}\frac{\partial \overline{H}^C}{\partial \overline{x}^C}}. \tag{A.2}$$

Following Reinganum (1985), we assume that the denominators of both expressions can be interpreted as a stability condition and, hence, are positive.[34]

The numerator of (A.1) is equal to

$$-h\left(\overline{x}^O\right)\left(h'\left(\overline{x}^C\right)\frac{L}{r} - 1\right)\left[\begin{array}{c} h''\left(\overline{x}^O\right)\left[L + \overline{x}^O + \left((R-C-1)\,h\left(\overline{x}^O\right) + Ch\left(\overline{x}^C\right)\right)\frac{L}{r}\right] \\ -h'\left(\overline{x}^O\right)\left(h'\left(\overline{x}^O\right)\frac{L}{r} - 1\right) \end{array}\right] > 0. \tag{A.3}$$

The numerator of (A.2) is equal to

$$h\left(\overline{x}^O\right)\left(h'\left(\overline{x}^O\right)\frac{L}{r} - 1\right)\left[\begin{array}{c} -h''\left(\overline{x}^C\right)\left(L + C\overline{x}^C + (R-C)\,h\left(\overline{x}^O\right)\frac{L}{r}\right) \\ +Ch'\left(\overline{x}^C\right)\left(h'\left(\overline{x}^C\right)\frac{L}{r} - 1\right) \end{array}\right] > 0. \tag{A.4}$$

Hence, we have $\frac{d\overline{x}^C}{dR}$ and $\frac{d\overline{x}^O}{dR} > 0$.

It follows that for any given outsider firm, $\overline{\alpha}_i = Ch\left(\overline{x}^C\right) + (R-C-1)\,h\left(\overline{x}^O\right)$ must also be increasing in R. Since by Lemma 1 the maximized profits of an outsider firm are decreasing in α_i, we can conclude that there exists a free-entry equilibrium where R^C denotes the number of participants in the race and all outsider participants earn zero profits.

[34] See p. 92 in Reinganum (1985).

2 Proof of Proposition 3

The first step is to show that $x^C < x^O$. Consider the first derivatives for the cartel's and a typical outsider's optimization problems. After imposing symmetry, these are given by

$$\widetilde{G}^C \equiv h'\left(\widetilde{x}^C\right)\left[L + C\widetilde{x}^C + \frac{L}{r}\left(R^C - C\right)h\left(x^O\right)\right] - \left[r + Ch\left(\widetilde{x}^C\right) + \left(R^C - C\right)h\left(x^O\right)\right] \tag{A.5}$$

and

$$\widetilde{H}^C \equiv h'\left(x^O\right)\left[L + x^O + \frac{L}{r}\left[\begin{array}{c}\left(R^C - C - 1\right)h\left(x^O\right)\\ +Ch\left(\widetilde{x}^C\right)\end{array}\right]\right] - \left[\begin{array}{c} r + Ch\left(\widetilde{x}^C\right) \\ +\left(R^C - C\right)h\left(x^O\right)\end{array}\right], \tag{A.6}$$

where x^O and $\widetilde{x}^C$ stand for the equilibrium investment level of an outsider firm and any symmetric investment level chosen by the cartel members, respectively. In equilibrium, $\widetilde{x}^C = x^C$. Note that

$$\frac{\partial\left(\widetilde{G}^C - \widetilde{H}^C\right)}{\partial \widetilde{x}^C} = h''\left(\widetilde{x}^C\right)\left[L + C\widetilde{x}^C + \frac{L}{r}\left(R^C - C\right)h\left(x^O\right)\right] - h'\left(\widetilde{x}^C\right)\left[Ch'\left(x^O\right)\frac{L}{r} - 1\right] < 0. \tag{A.7}$$

Moreover, $\widetilde{G}^C - \widetilde{H}^C$ evaluated at the point where $\widetilde{x}^C = x^O$ yields

$$-\left(C - 1\right)h'\left(x^O\right)\left[h\left(x^O\right)\frac{L}{r} - x^O\right] < 0. \tag{A.8}$$

Hence, whenever $\widetilde{G}^C - \widetilde{H}^C = 0$, which must be the case in equilibrium, we must have $x^C < x^O$.

We next show that $x^O = x^N$, which implies that if there are any active outsiders in the R&D race, each member of the R&D cartel invests $x^C < x^N$. The result follows because the first-order condition that an outsider firm in the presence of an R&D cartel solves is the same as the first-order condition that each active firm solves under R&D competition.

Each outsider maximizes the payoff function given in (2):

$$V_i\left(x_i, \alpha_i\right) = \frac{h\left(x_i\right)\frac{L}{r} - x_i}{r + h\left(x_i\right) + \alpha_i} - S$$

Taking the first derivative with respect to x_i and setting it equal to zero yields

$$h'\left(x_i\right)\left[L + x_i + \frac{L}{r}\alpha_i\right] - \left[r + h\left(x_i\right) + \alpha_i\right] = 0$$

where $\alpha_i = \sum_{j \neq i} h(x_j)$. In a symmetric equilibrium, the first order condition becomes

$$h'\left(x^O\right)\left[L + x^O + \frac{L}{r}\begin{pmatrix} \left(R^C - C - 1\right) h\left(x^O\right) \\ +Ch\left(x^C\right) \end{pmatrix}\right] - \begin{bmatrix} r + Ch\left(x^C\right) \\ +\left(R^C - C\right) h\left(x^O\right) \end{bmatrix} = 0 \qquad \text{(A.9)}$$

where R^C is determined by

$$\frac{h\left(x^O\right)\frac{L}{r} - x^O}{r + Ch\left(x^C\right) + \left(R^C - C\right) h\left(x^O\right)} - S = 0.$$

Solving this expression for R^C and substituting in (A.9) gives

$$h'\left(x^O\right)\left[L + x^O + \frac{L}{r}\left(\frac{h\left(x^O\right)\frac{L}{r} - x^O - rS - Sh\left(x^O\right)}{S}\right)\right] - \left[\frac{h\left(x^O\right)\frac{L}{r} - x^O}{S}\right] = 0. \quad \text{(A.10)}$$

Now consider the first-order condition of an active firm under R&D competition, which also maximizes (2). In a symmetric equilibrium, the first-order condition given in (3) simplifies to

$$h'\left(x^N\right)\left[L + x^N + \frac{L}{r}\left(R^N - 1\right) h\left(x^N\right)\right] - \left[r + R^N h\left(x^N\right)\right] = 0 \qquad \text{(A.11)}$$

where R^N is given by

$$\frac{h\left(x^N\right)\frac{L}{r} - x^N}{r + R^N h\left(x^N\right)} - S = 0.$$

Solving this expression for R^N and substituting in (A.11) yields

$$h'\left(x^N\right)\left[L + x^N + \frac{L}{r}\left(\frac{h\left(x^N\right)\frac{L}{r} - x^N - rS - Sh\left(x^N\right)}{S}\right)\right] - \left[\frac{h\left(x^N\right)\frac{L}{r} - x^N}{S}\right] = 0. \qquad \text{(A.12)}$$

Comparing this expression with (A.10) reveals that it must be the case that $x^O = x^N$.

It is insightful to note that the reason for this result follows from Lemma 1. Since both the active firms under R&D competition and outsiders in the presence of an R&D cartel face the profit function given by (2) and make zero profits in equilibrium, they must face the same value of α_i by Lemma 1. This implies that both type of firms choose the same investment level.

3 Proof of Proposition 4

(i) From the perspective of any outsider firm i, the aggregate arrival rate of innovation is equal to $h(x_i) + \alpha_i$. Due to free entry, the outsider firms earn zero profits both with an R&D cartel and under R&D competition:

$$\frac{h\left(x^N\right)\frac{L}{r} - x^N}{r + R^N h\left(x^N\right)} - S = \frac{h\left(x^N\right)\frac{L}{r} - x^N}{r + Ch\left(x^C\right) + \left(R^C - C\right) h\left(x^N\right)} - S = 0$$

Lemma 1 states that the profits of an outsider firm i change monotonically with α_i. Since the outsider firm i earns the same (zero) profits both with an R&D cartel and under R&D competition, this implies that a_i must be the same for the outsider firm i in each case. That is, $\left(R^N - 1\right) h\left(x^N\right) = Ch\left(x^C\right) + \left(R^C - C - 1\right) h\left(x^N\right)$. From Proposition 3, we know that the outsider firm invests x^N in both cases. Hence, we have $R^N h\left(x^N\right) = Ch\left(x^C\right) + \left(R^C - C\right) h\left(x^N\right)$. That is, the aggregate rate of innovation is the same in both cases.

(ii) From Proposition 3, we know that $x^C < x^N$. Since $R^N h\left(x^N\right) = Ch\left(x^C\right) + \left(R^C - C\right) h\left(x^N\right)$, this implies $R^C > R^N$.

4 Proof of Proposition 5

For an R&D cartel member, the aggregate hazard rate of its rival firms is $(C-1) h\left(x^C\right) + \left(R^C - C\right) h\left(x^N\right)$. For an outsider firm, the aggregate hazard rate of its rival firms is $Ch\left(x^C\right) + \left(R^C - C - 1\right) h\left(x^N\right)$. Since $x^C < x^N$, we have

$$(C-1) h\left(x^C\right) + \left(R^C - C\right) h\left(x^N\right) > Ch\left(x^C\right) + \left(R^C - C - 1\right) h\left(x^N\right).$$

By Lemma 1, this inequality implies that the outsider firm would earn a higher profit than the R&D cartel member if both firms maximized their individual profits. The R&D cartel member earns even less since it does not maximize its individual profits.

5 Proof of Proposition 6

Let $\overline{x}^J$ and $\overline{x}^O$ stand for the investment levels which satisfy (18) and (19), respectively, for a given number of cartel participants, J, and outsiders, $O = R - J$. We start by showing that $\overline{x}^J$ and $\overline{x}^O$ are both increasing in R by invoking a stability condition analogous to the one in the proof of Proposition 2.

For given values of J and R, let $\overline{G}^J$ and $\overline{H}^J$ represent the first-order conditions given in (18) and (19). $\overline{G}^J$ and $\overline{H}^J$ implicitly define $\overline{x}^J$ and $\overline{x}^O$. Totally differentiating and applying Cramer's Rule gives

$$\frac{d\overline{x}^J}{dR} = \frac{-\frac{\partial \overline{G}^J}{\partial R}\frac{\partial \overline{H}^J}{\partial \overline{x}^O} + \frac{\partial \overline{G}^J}{\partial \overline{x}^O}\frac{\partial \overline{H}^J}{\partial R}}{\frac{\partial \overline{G}^J}{\partial \overline{x}^J}\frac{\partial \overline{H}^J}{\partial \overline{x}^O} - \frac{\partial \overline{G}^J}{\partial \overline{x}^O}\frac{\partial \overline{H}^J}{\partial \overline{x}^J}} \tag{A.13}$$

and

$$\frac{d\overline{x}^O}{dR} = \frac{-\frac{\partial \overline{G}^J}{\partial \overline{x}^J}\frac{\partial \overline{H}^J}{\partial R} + \frac{\partial \overline{G}^J}{\partial R}\frac{\partial \overline{H}^J}{\partial \overline{x}^J}}{\frac{\partial \overline{G}^J}{\partial \overline{x}^J}\frac{\partial \overline{H}^J}{\partial \overline{x}^O} - \frac{\partial \overline{G}^J}{\partial \overline{x}^O}\frac{\partial \overline{H}^J}{\partial \overline{x}^J}}. \tag{A.14}$$

Following Reinganum (1985), we assume that the denominators of both expressions can be interpreted as a stability condition and, hence, are positive.[35]

The numerator of (A.13) is equal to

$$-h\left(\overline{x}^O\right)\left(Jh'\left(\overline{x}^J\right)\frac{L^J}{r} - 1\right)\left[\begin{array}{c} h''\left(\overline{x}^O\right)\left[L + \overline{x}^O + \frac{L}{r}\left((R-J-1)\,h\left(\overline{x}^O\right) + Ch\left(\overline{x}^J\right)\right)\right] \\ -h'\left(\overline{x}^O\right)\left(\frac{L}{r}h'\left(\overline{x}^O\right) - 1\right) \end{array}\right] > 0. \tag{A.15}$$

The numerator of (A.14) is equal to

$$Jh\left(\overline{x}^o\right)\left(h'\left(\overline{x}^O\right)\frac{L}{r} - 1\right)\left[\begin{array}{c} -h''\left(\overline{x}^J\right)\left[L^J + \overline{x}^J + \frac{L^J}{r}(R-J)\,h\left(\overline{x}^O\right)\right] \\ +h'\left(\overline{x}^J\right)\left(Jh'\left(\overline{x}^J\right)\frac{L^J}{r} - 1\right) \end{array}\right] > 0. \tag{A.16}$$

Hence, we have $\frac{d\overline{x}^J}{dR}$ and $\frac{d\overline{x}^O}{dR} > 0$.

It follows that for any given outsider firm, $\overline{\alpha}_i = Jh\left(\overline{x}^J\right) + (R-J-1)\,h\left(\overline{x}^O\right)$ must also be increasing in R. Since by Lemma 1 the maximized profits of an outsider firm are decreasing in α_i, we can conclude that there exists a free-entry equilibrium where R^J denotes the number of participants in the race and all outsider participants earn zero profits.

6 Proof of Lemma 2

The free-entry equilibrium investment levels and number of firms are implicitly defined by (18), (19), and (20). Let G^J, H^J, and Z^J stand for these three conditions. Totally

[35] See p. 92 in Reinganum (1985).

differentiating and applying Cramer's Rule gives us

$$\frac{dx^J}{dL^J} = \frac{\frac{\partial G^J}{\partial L^J}\left[\frac{\partial Z^J}{\partial x^O}\frac{\partial H^J}{\partial R} - \frac{\partial Z^J}{\partial R}\frac{\partial H^J}{\partial x^O}\right]}{\frac{\partial Z^J}{\partial x^J}\left[\frac{\partial G^J}{\partial x^O}\frac{\partial H^J}{\partial R} - \frac{\partial G^J}{\partial R}\frac{\partial H^J}{\partial x^O}\right] - \frac{\partial Z^J}{\partial x^O}\left[\frac{\partial G^J}{\partial x^J}\frac{\partial H^J}{\partial R} - \frac{\partial G^J}{\partial R}\frac{\partial H^J}{\partial x^J}\right] + \frac{\partial Z^J}{\partial R}\left[\frac{\partial G^J}{\partial x^J}\frac{\partial H^J}{\partial x^O} - \frac{\partial G^J}{\partial x^O}\frac{\partial H^J}{\partial x^J}\right]}. \tag{A.17}$$

The stability condition implies that $\frac{\partial G^J}{\partial x^J}\frac{\partial H^J}{\partial x^O} > \frac{\partial G^J}{\partial x^O}\frac{\partial H^J}{\partial x^J}$. Since $\frac{\partial G^J}{\partial x^J} < 0$, $\frac{\partial G^J}{\partial x^O} > 0$, and $\frac{\partial H^J}{\partial x^J} > 0$, we must have $\frac{\partial H^J}{\partial x^O} < 0$. Furthermore, $\frac{\partial G^J}{\partial L^J}$, $\frac{\partial H^J}{\partial R}$, $\frac{\partial G^J}{\partial R}$, $\frac{\partial G^J}{\partial x^O}$ and $\frac{\partial H^J}{\partial x^J}$ are > 0 while $\frac{\partial G^J}{\partial x^J}$, $\frac{\partial H^J}{\partial x^O}$, $\frac{\partial Z^J}{\partial R}$, $\frac{\partial Z^J}{\partial x^O}$ and $\frac{\partial Z^J}{\partial x^J}$ are < 0. Hence, both the numerator and denominator of (A.17) are negative, and we have $\frac{dx^J}{dL^J} > 0$.

To prove that equilibrium RJV cartel profits are monotonically increasing in L^J, note

$$\frac{d\left(JV^J\right)}{dL^J} = \frac{\partial\left(JV^J\right)}{\partial L^J} + \frac{\partial\left(JV^J\right)}{\partial \alpha^J}\frac{\partial \alpha^J}{\partial L^J}, \tag{A.18}$$

where V^J stands for the per-firm profit level with an RJV cartel and $\alpha^J = \left(R^J - J\right)x^O$. The first term on the right hand side is positive and the first part of the second term is negative by inspection of (16). From Lemma 1 we know that in a free-entry equilibrium, the outsiders must face the same value of α_i regardless of the value of L^J. This implies that $\frac{dR^J}{dL^J} < 0$ since $x^O = x^N$ and $\frac{dx^J}{dL^J} > 0$, as established above. Hence, the second part of the second term is negative also.

7 Proof of Lemma 3

Substituting for $L^J = \frac{L}{J}$ in the first derivative of (12) with respect to x_i reveals that if $C = J$, i.e., if an R&D cartel and an RJV cartel both have the same number of firms, the per-firm investment level is the same under both types of cooperative arrangements. Similarly, substituting for $L^J = \frac{L}{J}$ in the equilibrium payoff level shows that the profits are also the same under the two types of cooperative arrangements. Hence, the results for $L^J = \frac{L}{J}$ follow from Proposition 3 and Proposition 5.

Consider now the case where $L^J = L$. The first step is to show that $x^J > x^O$. Consider the first derivatives for the cartel's and a typical outsider's optimization problems. After imposing symmetry, these are given by

$$\widetilde{G}^J \equiv Jh'\left(\widetilde{x}^J\right)\left[L^J + \widetilde{x}^J + \frac{L^J}{r}\left(R^J - J\right)h\left(x^O\right)\right] - \left[r + Jh\left(\widetilde{x}^J\right) + \left(R^J - J\right)h\left(x^O\right)\right] \tag{A.19}$$

and

$$\widetilde{H}^J \equiv h'\left(x^O\right)\left[L + x^O + \frac{L}{r}\left[\begin{array}{c}\left(R^J - J - 1\right)h\left(x^O\right)\\ +Jh\left(\widetilde{x}^J\right)\end{array}\right]\right] - \left[\begin{array}{c} r + Jh\left(\widetilde{x}^J\right)\\ +\left(R^J - J\right)h\left(x^O\right)\end{array}\right], \tag{A.20}$$

where x^O and $\widetilde{x}^J$ stand for the equilibrium investment level of an outsider firm and any symmetric investment level chosen by the cartel members, respectively. In equilibrium, $\widetilde{x}^J = x^J$. Note that

$$\frac{\partial\left(\widetilde{G}^J - \widetilde{H}^J\right)}{\partial \widetilde{x}^J} = Jh''\left(\widetilde{x}^J\right)\left[L + \widetilde{x}^J + \frac{L}{r}\left(R^J - J\right)h\left(x^O\right)\right] - Jh'\left(\widetilde{x}^J\right)\left[h'\left(x^O\right)\frac{L}{r} - 1\right] < 0. \tag{A.21}$$

Moreover, $\widetilde{G}^J - \widetilde{H}^J$ evaluated at the point where $\widetilde{x}^J = x^O$ yields

$$(J-1)\,h'\left(x^O\right)\left[L + x^O + \frac{L}{r}\left(R^J - J - 1\right)h\left(x^O\right)\right] > 0. \tag{A.22}$$

Hence, whenever $\widetilde{G}^J - \widetilde{H}^J = 0$, which must be the case in equilibrium, we must have $x^J > x^O$.

We next show that any active outsider participant in the R&D race must invest x^N. All active outsider firms in the R&D race earn zero profits in equilibrium. By Lemma 1, this implies that an outsider firm i must face the same value of α_i as it does under R&D competition. Hence, it solves the same maximization problem as it does under R&D competition and invests x^N. This result together with the analysis above implies that if there are any active outsider participants in the R&D race, each member of the RJV cartel invests $x^J > x^N$ in equilibrium.

To see that the RJV cartel earns positive profits, note that if we hold the outsiders' investments constant at x^N and decrease the RJV cartel's investment to x^N, the RJV cartel's per firm profits are

$$\frac{h\left(x^N\right)\frac{JL}{r} - x^N}{r + Jh\left(x^N\right) + \left(R^J - J\right)h\left(x^N\right)} - S \tag{A.23}$$

and an outsider firm earns

$$\frac{h\left(x^N\right)\frac{L}{r}-x^N}{r+Jh\left(x^N\right)+\left(R^J-J\right)h\left(x^N\right)}-S, \tag{A.24}$$

which is clearly less. However, the outsider firm would be earning strictly positive profits, since there would be fewer firms in total making the same per-firm investments as under R&D competition. Hence, the RJV cartel would also be making strictly positive profits. Since the RJV cartel chooses x^J to maximize its joint profits given the outside firms choose x^N, it must earn even higher profits in equilibrium.

8 Proof of Proposition 7

The existence of $\widehat{L}^J \in \left(\frac{L}{J}, L\right)$ and $\widetilde{L}^J \in \left(\frac{L}{J}, L\right)$ follow from Lemmas 3 and 3. To prove that $\widetilde{L}^J > \widehat{L}^J$, we evaluate the profitability of an RJV cartel when $L^J = \widetilde{L}^J$ and show that it is positive. When $L^J = \widetilde{L}^J$, the RJV cartel's equilibrium per-firm investment is x^N by definition. Note that each outsider participant in the R&D race in equilibrium earns

$$\frac{h\left(x^N\right)\frac{L}{r}-x^N}{r+Jh\left(x^N\right)+\left(R^J-J\right)h\left(x^N\right)}-S=0 \tag{A.25}$$

while each member of the RJV cartel earns

$$\frac{h\left(x^N\right)\frac{J\widetilde{L}^J}{r}-x^N}{r+Jh\left(x^N\right)+\left(R^J-J\right)h\left(x^N\right)}-S. \tag{A.26}$$

Subtracting (A.25) from (A.26) yields

$$\frac{h\left(x^N\right)\left[\frac{J\widetilde{L}^J-L}{r}\right]}{r+Jh\left(x^N\right)+\left(R^J-J\right)h\left(x^N\right)}>0 \tag{A.27}$$

since $\widetilde{L}^J > L$.

9 Proof of Proposition 9

We present the proof for the case of an R&D cartel only since the case of an RJV cartel is identical. Let $R^N_{no}h\left(x^N_{no}\right)$ stand for the aggregate rate of innovation under R&D competition and $C_{no}h\left(x^C_{no}\right)$ stand for the aggregate rate of innovation with an R&D cartel. The subscript *no* refers to the case of no outsiders. We would like to show that $C_{no}h\left(x^C_{no}\right) \geqslant R^N_{no}h\left(x^N_{no}\right)$.

Suppose not. That is, suppose there are no outsiders in the R&D race in equilibrium and $C_{no}h\left(x_{no}^{C}\right) < R_{no}^{N}h\left(x_{no}^{N}\right)$. Consider the marginal entrant to the R&D race under R&D competition. The aggregate hazard rate of its rivals is given by $R_{no}^{N}h\left(x_{no}^{N}\right)$. For a potential entrant to the R&D race with an R&D cartel, the aggregate hazard rate of its rivals, given by $C_{no}h\left(x_{no}^{C}\right)$, is lower. Hence, by Lemma 1, it would find it profitable to enter the market. Since this violates one of the conditions for equilibrium, it cannot be the case that $C_{no}h\left(x_{no}^{C}\right) < R_{no}^{N}h\left(x_{no}^{N}\right)$ in equilibrium.